CodPy : a Python library for machine learning, mathematical finance, and statistics

Philippe G. LeFloch[1] , Jean-Marc Mercier, and Shohruh Miryusupov[2]

2021-06-04

Activities : MPG Partners is a mid-sized French consulting firm specialized in risk management : ~ 50 employees / 22 clients

BPCE BNP PARIBAS bpifrance CA CIB Caisse des Dépôts CA SFIL NATIXIS LA BANQUE POSTALE ASSET MANAGEMENT

Small size R&D team

- Jean-Marc Mercier jean-marc.mercier@mpg-partners.com
- Shohruh Miryusupov shohruh.miryusupov @mpg-partners.com

cnrs

Philippe LeFloch
contact@philippelefloch.org

SORBONNE UNIVERSITÉ

MPG PARTNERS

CodPy is a python library

kernel-based (RKHS - Reproducing Kernel Hilbert Space).

C++ core / python or graph database (xquery / xml based) interface

- Compete with

Tensorflow, Pytorch, Theano, scikit-learn, xgboost, ADA boost, k-trees, Random Forest,...)

(Artificial Intelligence, Neural Networks, Deep Learning, Support Vector Machine, k-trees, etc...)

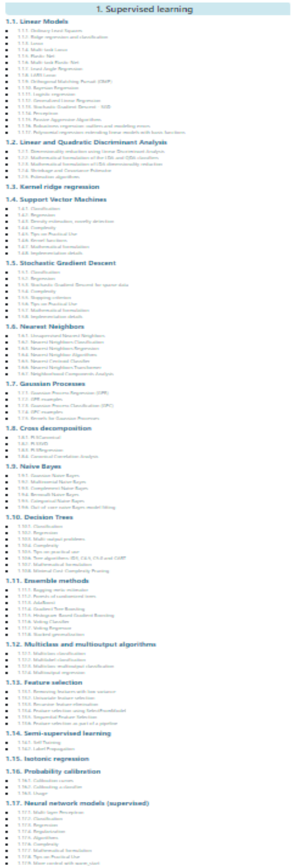
- **Users** : 2014 - today **: internal library.** Tomorrow **: public access ? (pip install CodPy) ?**

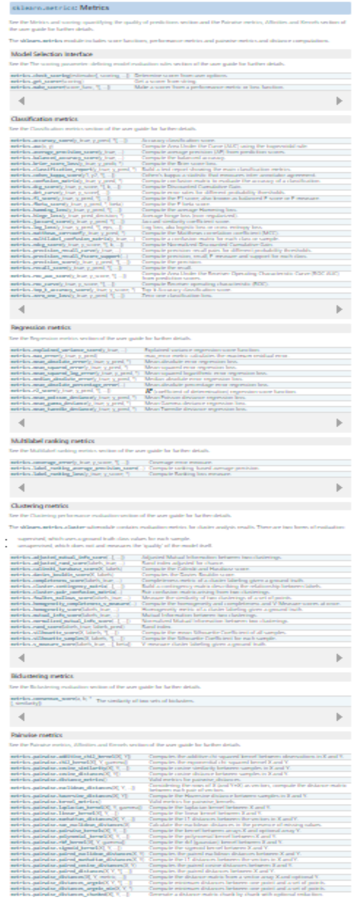
- **Our user manuals are publicly available** (see last slide - bibliography) supervised learning methods

unsupervised learning methods

Performance indicators

Librairies





List of IA librairies on PiPy

3,815 projects with the selected classifier

TOPIC :: SCIENTIFIC/ENGINEERING :: ARTIFICIAL INTELLIGENCE X

+1:
kernel Projection
(multi-output / labelled-unlabelled supervised learning machine)

+1:
Sharp discrepancy sequences
(clustering method)

+2:
Discrepancy error
Functional norms

+1:
CodPy

MPG PARTNERS

supervised learning methods :
A methodology to benchmark them all

$$\textbf{Prediction function } f_z = P(x, y, z, f(x))$$

- $x \in \mathbb{R}^{N_x \times D}$ training set.
- $y \in \mathbb{R}^{N_y \times D}$ **weight set.**
- $z \in \mathbb{R}^{N_z \times D}$ test set.
- $f(x) \in \mathbb{R}^{N_x \times D_f}$ multi-valued

STEP I: pick up some libraries, and start setting their inner parameters

Neural Networks settings

```
import tensorflow as tf
tf_param = {'epochs': 10,
'batch_size':16,
'validation_split':0.1,
'loss': tf.keras.losses.SparseCategoricalCrossentropy(from_logits=True),
'optimizer':tf.keras.optimizers.Adam(0.001),
'activation':['relu',''],
'layers':[128,10],
'metrics':[tf.keras.metrics.SparseCategoricalAccuracy()]}
```

Random Forest settings

```
RF_param = {'max_depth': 5,
'n_estimators': 5}
```

Xgboost settings

```
xgb_param = {'max_depth': 5,
    'n_estimators': 10,
    'num_class' : 10,
    'objective':'multi:softmax',
    'eta': 0.3,
    'min_child_weight' : 1,
    'num_boost_round': 100,
    'eval_metric': 'mlogloss',
}
```

AdaBoost settings

```
ada_param = {'tree_no': 50,
'base_model':'tree_model',
'learning_rate': 1,
'algorithm':'SAMME.R'}
```

CodPy settings

```
codpy_param = {'set_kernel': 'gaussian'}
```

MPG PARTNERS

supervised learning methods : the MNIST database (Y. Le Cun 1998)

- **II-a pick-up a data set.** MNIST (pattern recognition problem)
- **II-b Pick-up performance indicators :** accuracy scores, discrepancy errors
- **II-c Pick-up a list of tests :** variable training set size **(Nx)**

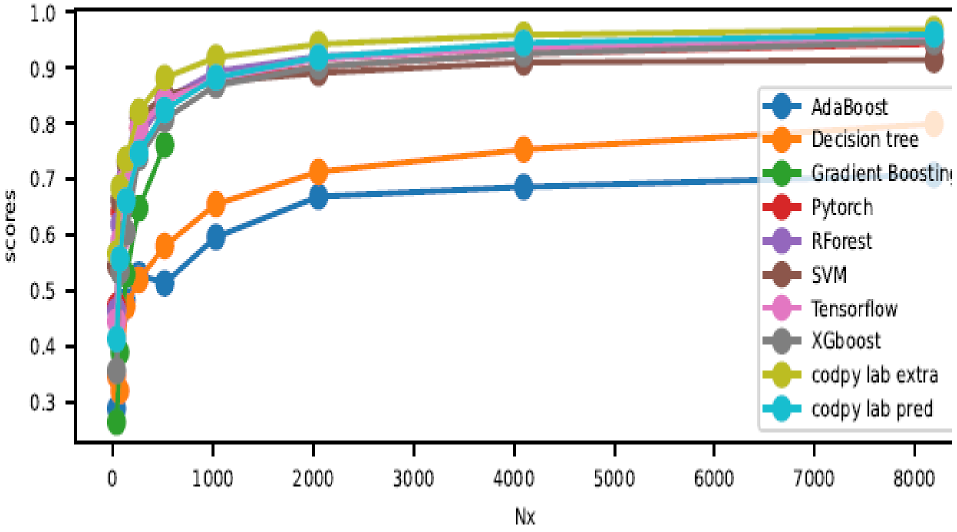


Figure 5.2: Benchmark scores

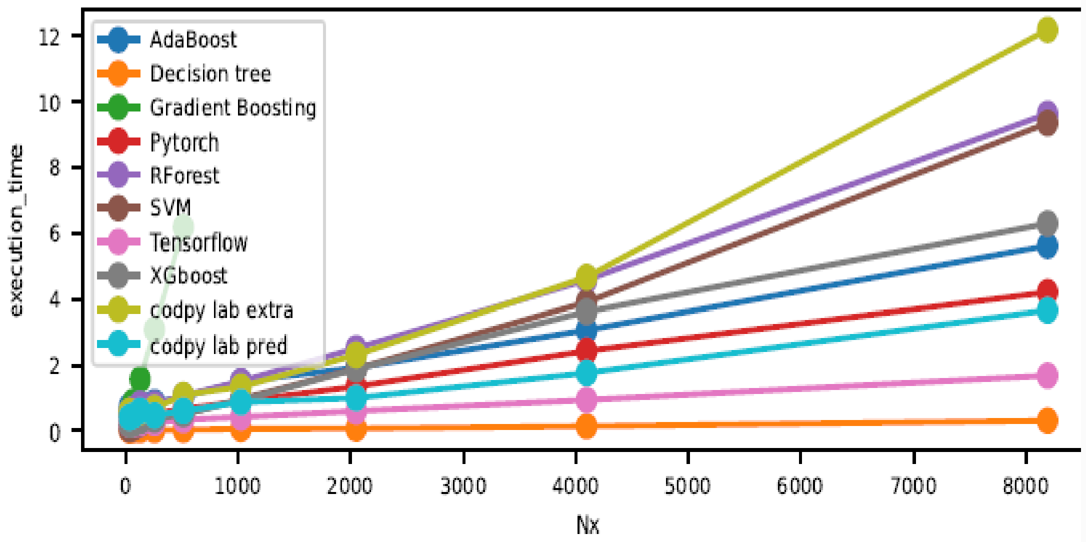


Figure 5.4: Benchmarks execution time

What differentiate kernel methods from other machine learning technics ?

Error estimates

$$\|f(z) - f_z\| \leq d_k(x, y, z)\|f\|_{H_k}$$

- $d_k(x, y, z)$ discrepancy error.
- $\|f\|_{H_k}$ function norm.

Both python functions

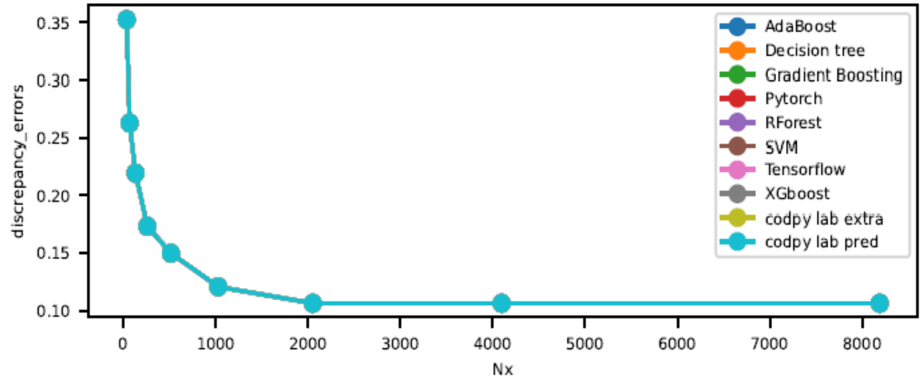


Figure 5.3: Discrepancy errors

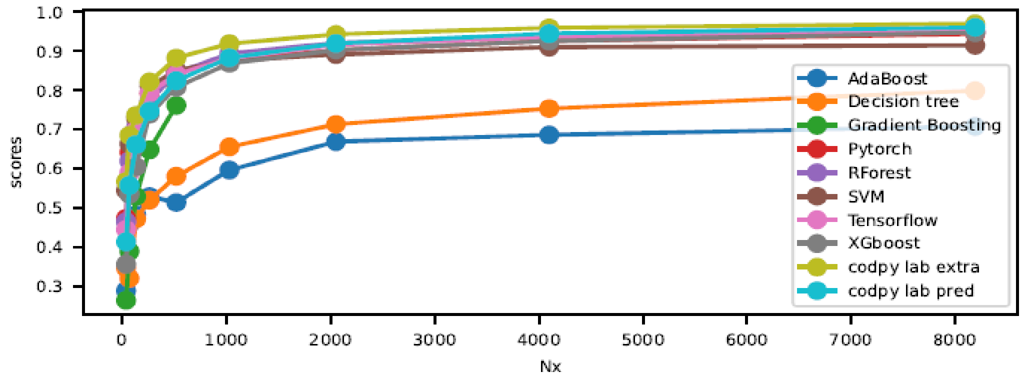


Figure 5.2: Benchmark scores

Kernel-based scores for MNIST can be **predicted** by discrepancy errors.
Codpy score <~ 1 – d(x,y,z)

What differentiate kernel methods from other machine learning ?

Error estimates Part II

$$\|f(z) - f_z\| \leq d_k(x, y, z)\|f\|_{H_k}$$

- $d_k(x, y, z)$ discrepancy error.
- $\|f\|_{H_k}$ function norm.

Each kernel-based learning machine is shipped with quantified, worst-error analysis tools for supervised / unsupervised learning

Consequence 1 : kernel-based methods are **CONVERGENT**

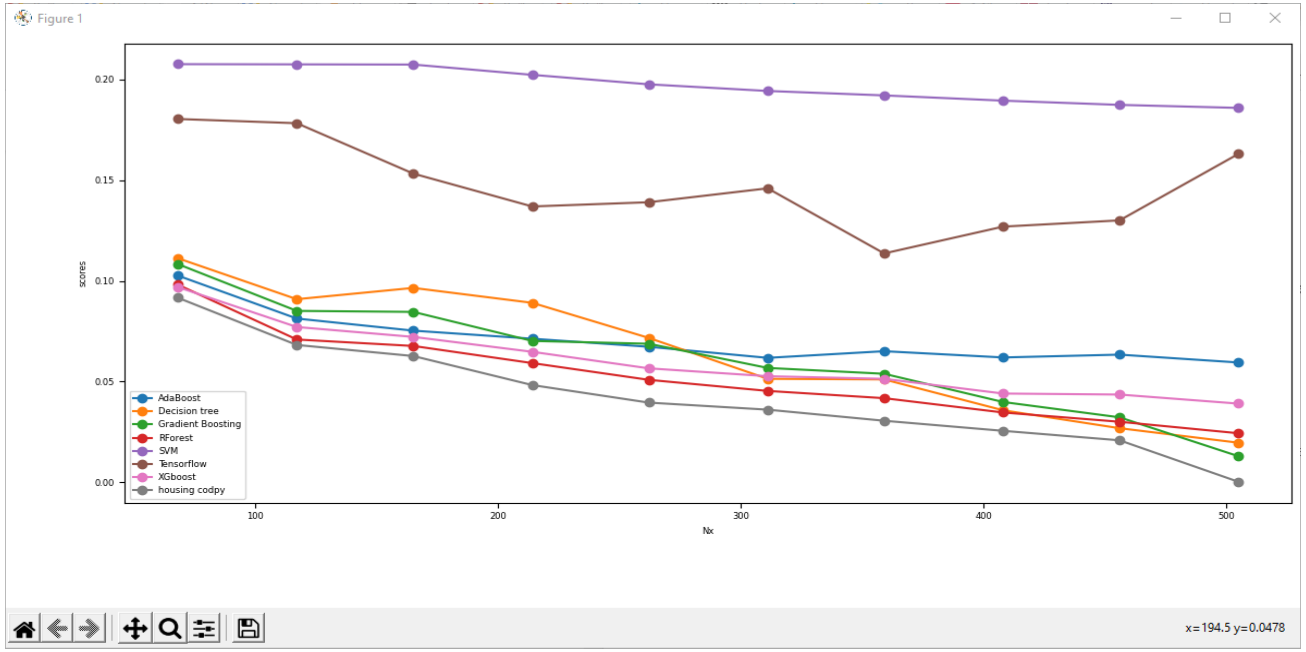
Consequence 2 : kernel-based methods are **EXPLAINABLE, hence AUDITABLE**

Another benchmark: the venerable **BOSTON Housing price (1970)** **(market price prediction)**

- **II-a pick-up a data set.** BOSTON Housing prices
- **II-b Pick-up performance indicators :** Round Mean Square error %
- **II-c Pick-up a list of tests :** variable training set size (Nx)

Test set =all Boston database (506 entries)

the training set is part of the test set



Note:

score of the kernel method is zero on the last point (perfect score) :

kernel methods can replicate exactly the training set.

Reconstruction from sub-sampled signals.

medical imagery, oceanography, petroleum, defense, astrophysics …

Application example: reconstruction from sub-sampled signals for SPECT machine

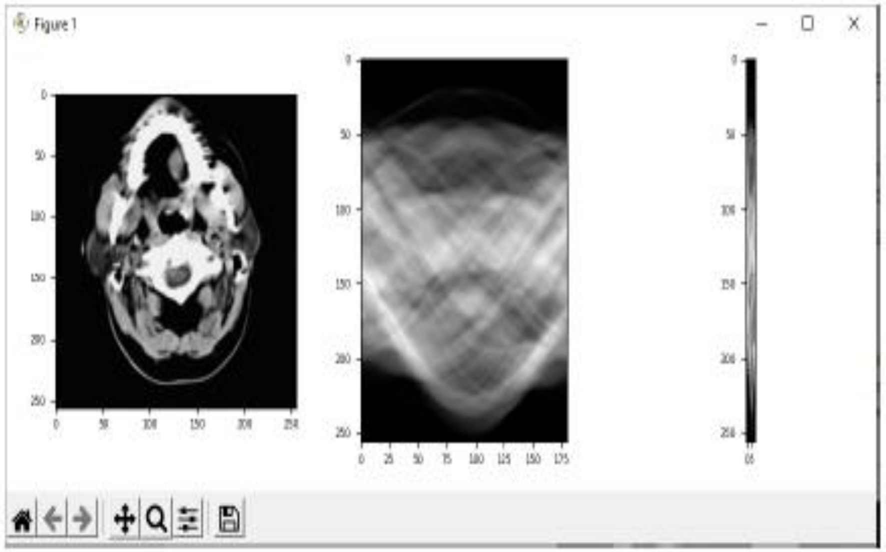


Figure 5.5: high resolution sinogram (middle), low resolution (right), reconstructed image (left)

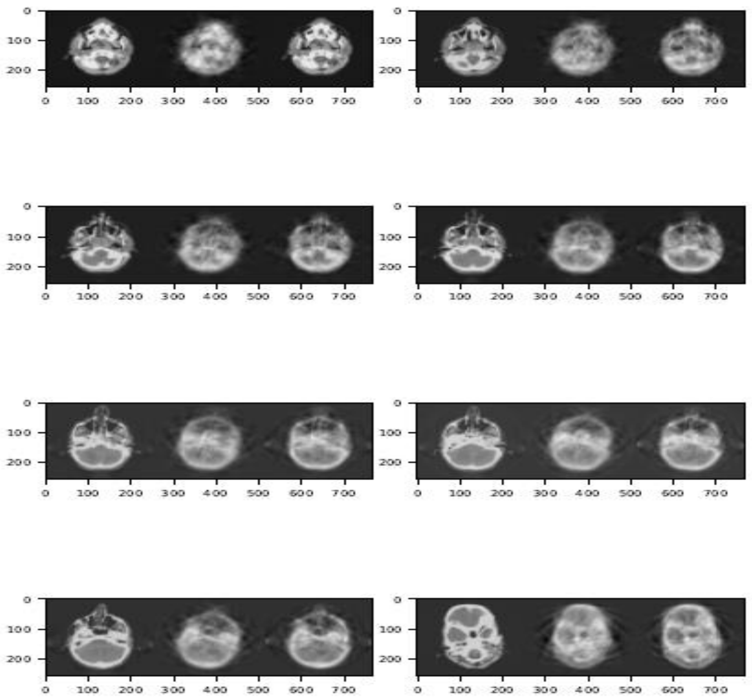


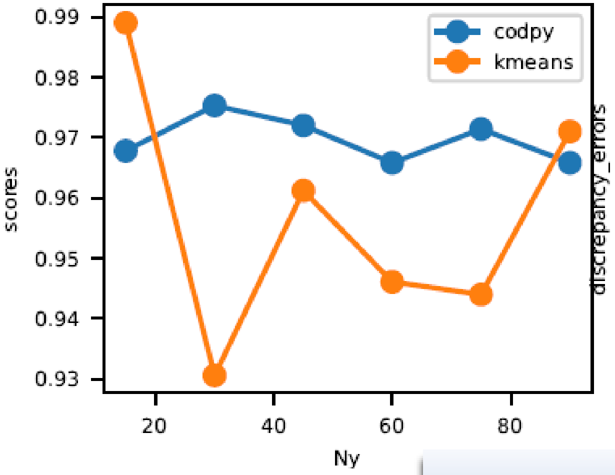
Figure 5.6: Reconstructing from sub-sampled datas. Left original. middle SART method. right kernel extrapolation.

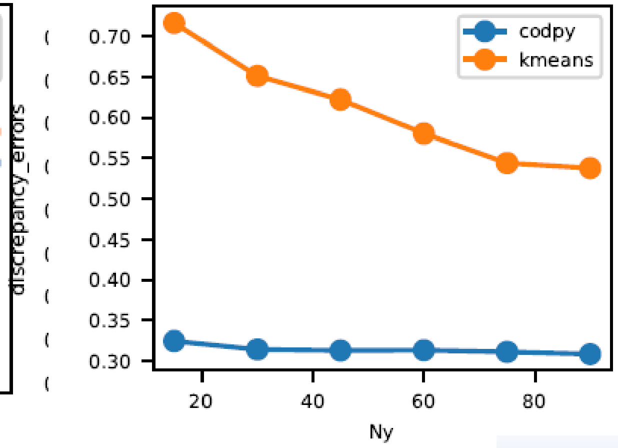
Unsupervised learning methods (clustering / segmentation)
a methodology to benchmark them all

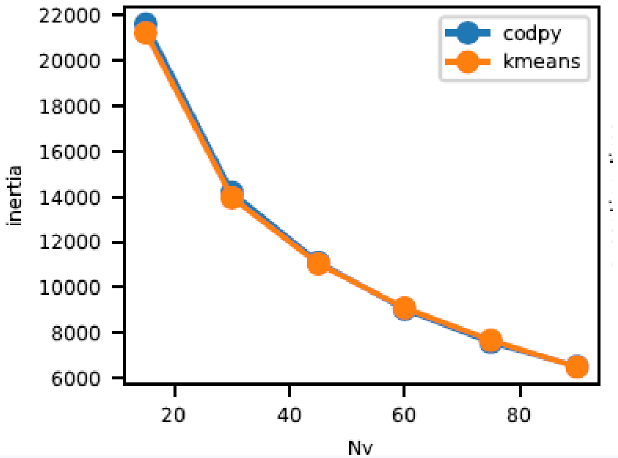
$$\text{Prediction function } y = P(x, N_y)$$

- $x \in \mathbb{R}^{N_x \times D}$ training set.
- N_y number of clusters.
- $y \in \mathbb{R}^{N_y \times D}$ clusters (segmentation / quantization)

Benchmark: k-means algorithm versus sharp discrepancy sequences
DataBase : Kaggle credit card fraud detection (284 00 entries / 500 frauds)
Performance indicators : scores, discrepancy errors, inertia







Note:
Kernel scores are stables

Why ? Discrepancy sequences are deterministics
K-means clusters are not (initial values).

MPG PARTNERS

What differentiate kernel methods from other machine learning ?

Differential operators

RKHS is a theory of functional spaces

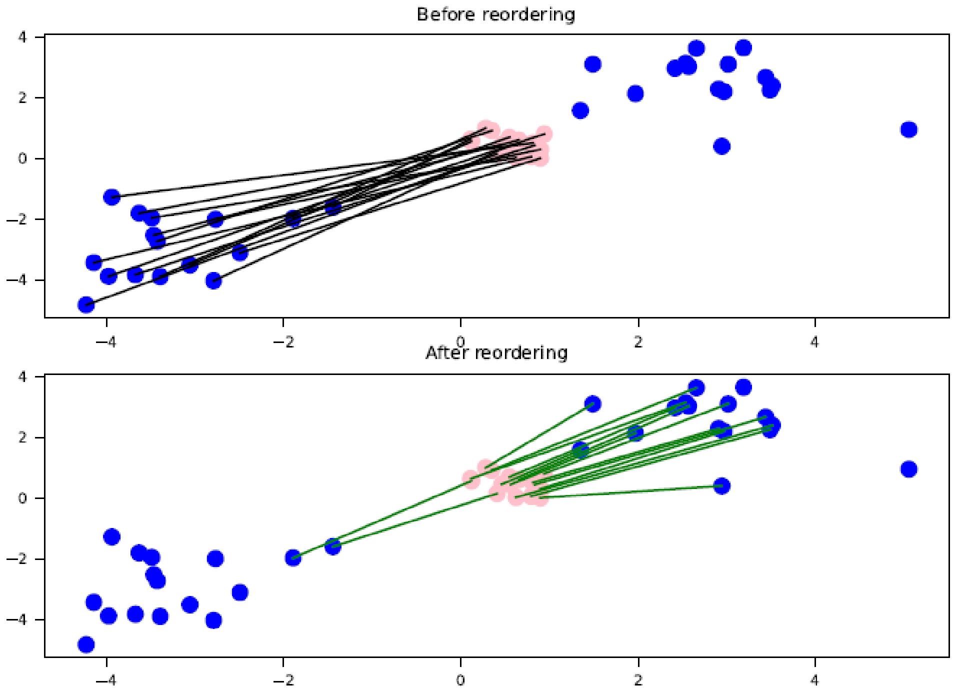
Kernel-based learning machines come with **Differential / integral operators**

Every kernel learning machine has access to **OPTIMAL TRANSPORTATION** algorithms

MPG PARTNERS

Optimal Transportation : Monge-Kantorovitch and kernel-based reordering algorithms

$$\inf_\gamma D_k(x,z)\cdot\gamma, \quad \gamma \text{ bi-stochastic}, D_k(x,z) \text{ discrepancy errors}$$



MPG PARTNERS

Optimal Transportation :
Polar factorization algorithms

Polar Factorization: find h **convex** s.t. $z = (\nabla h) \circ T(x)$

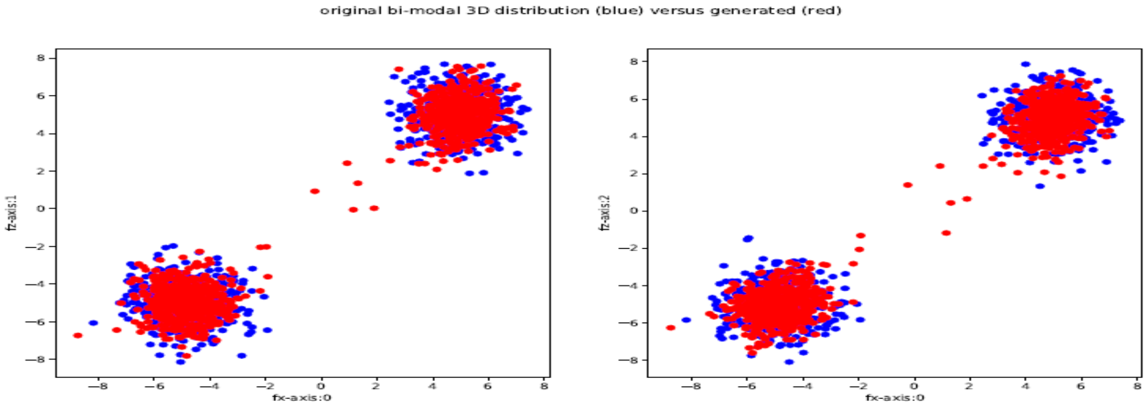
Application : the sampling function $z = S_k(x, N_z)$

Inputs: k **kernel**

- $x \in \mathbb{R}^{N \times D}$ iid of X.
- N_z number of samples.

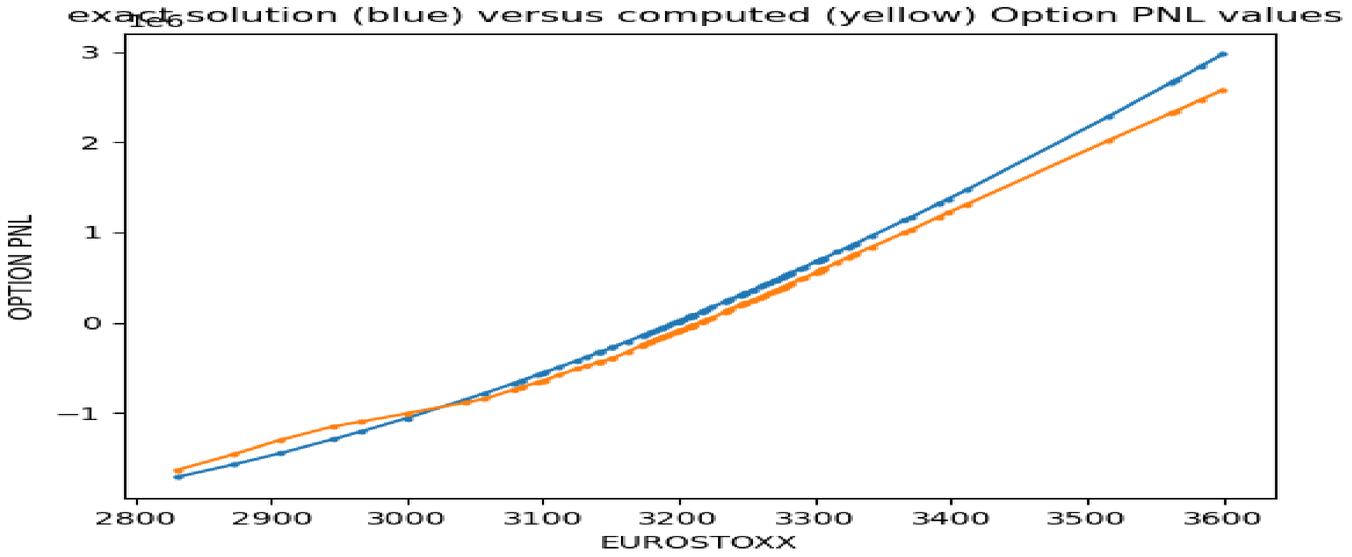
Output: $z \in \mathbb{R}^{N_z \times D}$,
a distribution **sharing close statistical properties** with x

Example with a multi-dimensional bi-modal distribution



Polar factorization algorithms applications:
reconstruction of time dependent functions from time series observations
Stress test and reverse stress test

1 - Observe a time serie $X_{t<0}$ (market observations)
2 - Observe a time serie $P\&L(X_{t<0})$ (P&L observations)
3 - today $= t = 0$. Set $T > 0$ an Horizon
4 - Use $S_k(X_t, N_z)$ to reproduce the distribution $P\&L(X_T)$ (stress test)
5 - Use $S_k(P\&L(X_t), N_z)$ to find scenari X_T (reverse stress test)
6 - Use error estimates to produce confidence level on risk measurements



Note (reverse stress test)

discrepancy errors are more meaningful than Mahalanobis distance for kernel methods

MPG PARTNERS

Polar factorization algorithms applications: Transition probability / conditional expectations

Transition probability operator $f_{z|x} = \Pi(x, z, f(z), k)$

Consider $t \mapsto X_t$ any **martingale** process.

Inputs: k **kernel**

- $x \in \mathbb{R}^{N \times D}$ iid of X_{t_1}.
- $z \in \mathbb{R}^{N \times D}$ iid of X_{t_2}, $t_2 > t_1$.
- $f(z) \in \mathbb{R}^{N \times D_f}$ optional function values.

Output: transition probabilities operator

- $f_{z|x} \sim \mathbb{E}^{X_{t_2}}(f(\cdot)|X_{t_1} = x) \in \mathbb{R}^{N \times D_f}$.
- $\Pi(x, z) = p(z^n|x^m)_{n=1..N}^{m=1..N} \in \mathbb{R}^{N \times N}$.

  stochastic matrix - probabilities of transition.

Straightforward applications

- **Pricing / risk (eg XVA) with internally generated samples**
- **alternative to Bayesian classifiers**

Benchmark of conditional expectations algorithms : the Bachelier problem

Consider $t \mapsto X_t \in \mathbb{R}^D$ a Brownian process (random parameters).

Inputs: k kernel

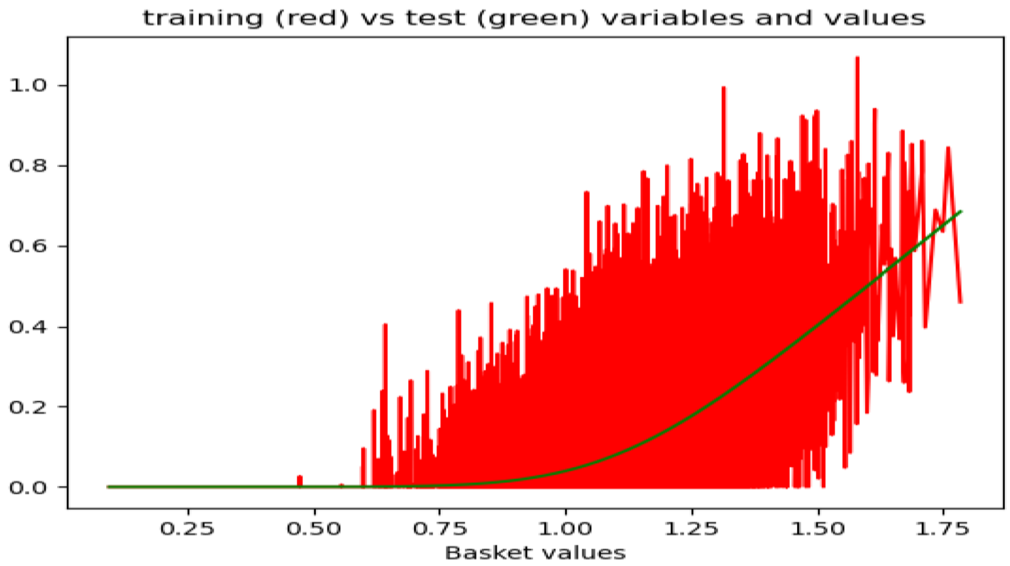
- $x \in \mathbb{R}^{N \times D}$ iid of $X_{t=1}$.
- $z \in \mathbb{R}^{N \times D}$ iid of $X_{t=2}$.
- $f(z) := (z^T a - K)^+$ option payoff, a random weights

Output: forward values of options

- $f_{z|x} \sim \mathbb{E}^{X_{t=2}}(f(\cdot)|X_{t=1} = x)$ computed.

benchmarked against
$f(z|x) := \mathbb{E}^{X_{t=2}}(f(\cdot)|X_{t=1} = x)$ - closed formula.

We want to deduce the green curve $f(z|x)$ from the red data $f(z)$

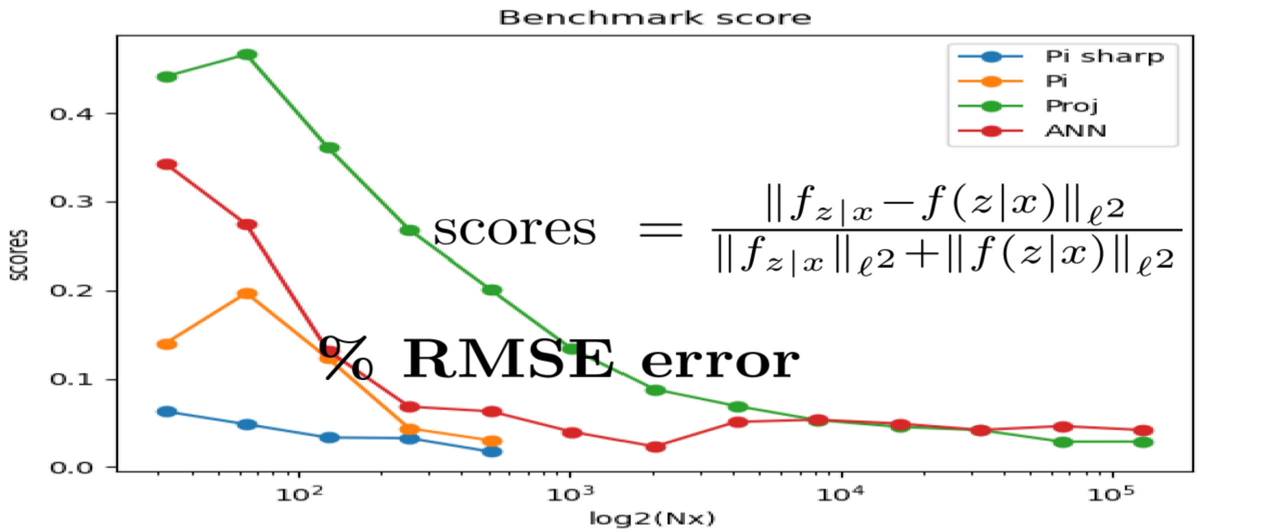


MPG PARTNERS

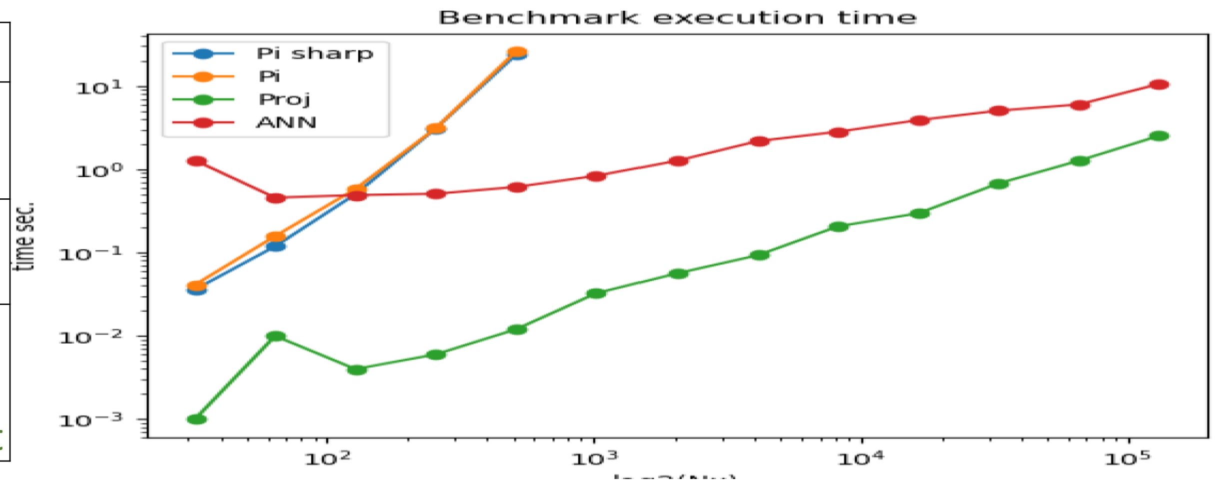
Kernel methods - Tools for statistical Learning
the Bachelier problem : a toy example of audit

Score and time Benchmark :

- **ANN = Tensorflow**
- **Proj = CodPy P(x,y,z,fx,k)**
- **Pi = CodPy Pi(x,z,fx,k)**
- **Pi sharp = CodPy Pi(x,z,fx,k)**
  (x,z sharp discrepancy sequences)



method	Sharp	Pi	ANN	Proj
size of training set	64	512	100 000	100 000
error % RMSE	3.40%	3.00%	4.10%	2.80%
diagnostic	convergent	convergent	**NOT** convergent	**NOT** convergent



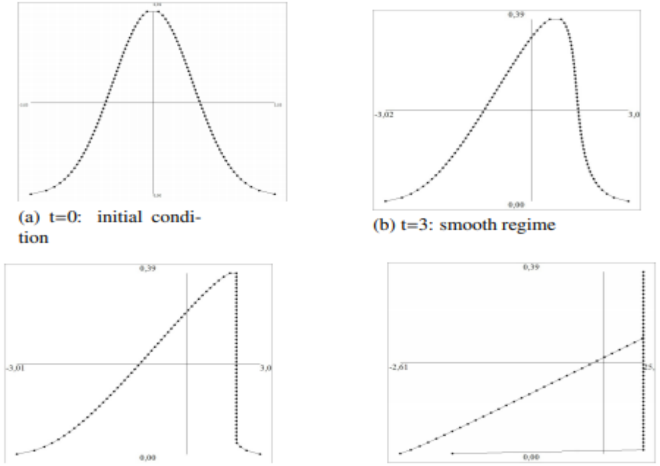
MPG PARTNERS

Optimal Transportation :

Signed Polar factorization algorithms and the convex-hull algorithms (CHA) explicit solutions to Hamilton-Jacobi equations

CHA algorithm:

- Decompose $z = (\nabla h) \circ T(x)$, T "positive".
- Consider $z^+ = (\nabla h^+) \circ T(x)$, h^+ convex hull of h.



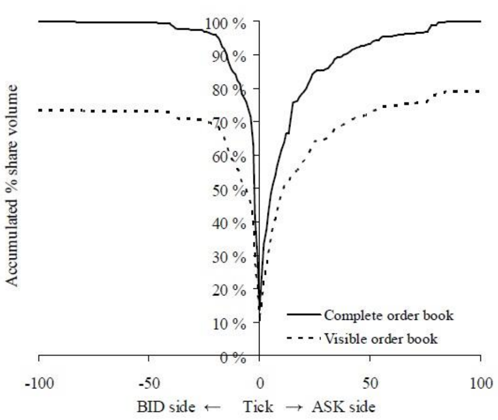
(a) t=0: initial condition

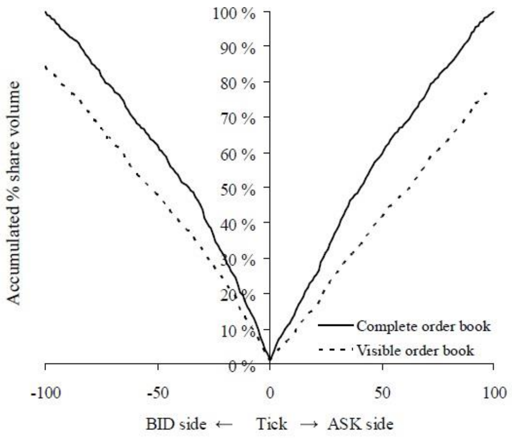
(b) t=3: smooth regime

(c)t=6: shock formation

(d) t=106: N-wave regime.

Figure 4.1: Entropy dissipative solutions to Burgers equation





MPG PARTNERS

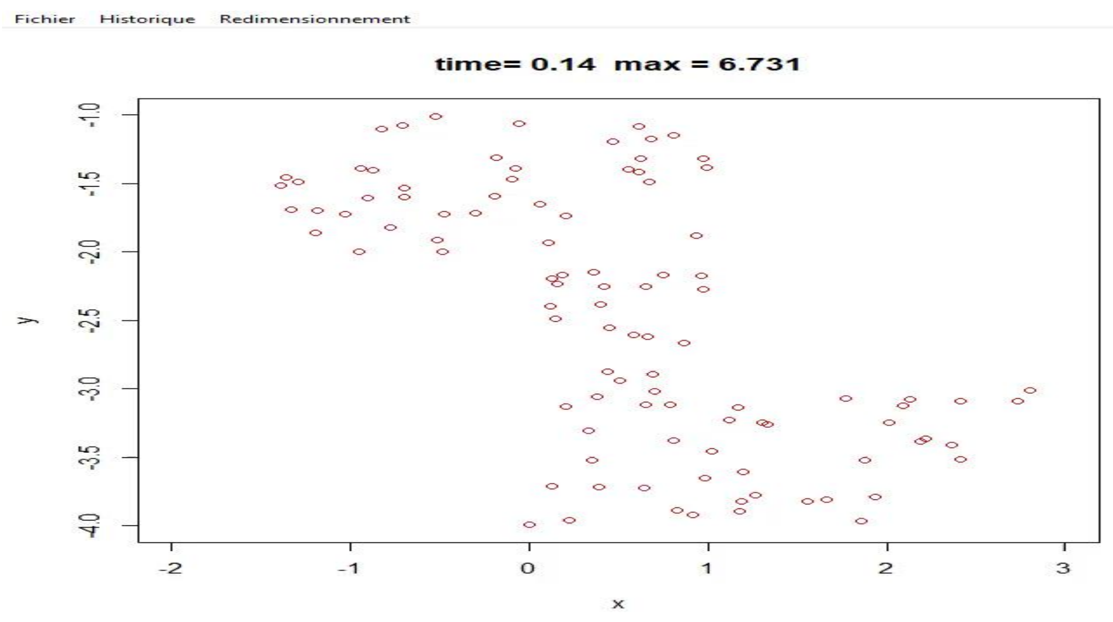
What differentiate kernel methods from other machine learning ?

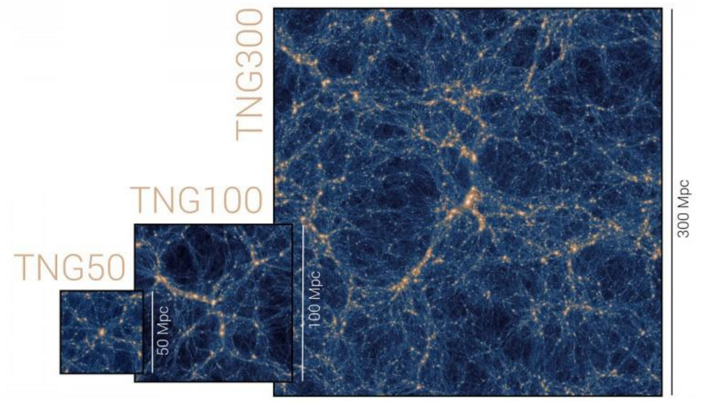
Differential operators / sharp discrepancy sequences

Transported meshfree methods

Share similarities with

smooth particle hydrodynamic





Apps:
Video games
Astronomical simulations
…

MPG PARTNERS

What are kernel methods for numerical simulations in Finance ?

High-dimensional **Hamilton-Jacobi-Bellman** PDE solvers

Front Office / Risk management engines since 2012

$$dX_t = r(t, X_t)dt + \sigma(t, X_t)dW_t$$
$$P(t, X_t, \nabla g(X_t), \ldots)$$

Any kind of stochastic process, any dimensions

Any kind of function (payoff / strategy optimal control)

1. Any kind of forward / backward modeling (**BSDE/FSDE**).
2. **PDE** (Partial Differential Equation) solver engine **Fokker-Plank / Kolmogorov (Forward / Backward)**
3. Huge portfolios / risks sources capabilities.
4. Bullet proof **auditable** due to **worst-error bounds**.
5. Any kind of risk measurements (from instant prices to **forward greeks**).
6. **Real time** versus **accuracy** easy tuning capabilities.

Apps:
Pricing / Risk / XVA / investment strategies / arbitrage / etc etc …

MPG PARTNERS

Want to dig in ? Our bibliography

:

theoretical

We propose novel methods for machine learning and numerical simulations, using a partial differential equations approach.

1) *A class of mesh-free algorithms for mathematical finance, machine learning and fluid dynamics. This paper is the backbone of our approach.*
2) *"The Transport-based Mesh-free Method (TMM) and its applications in finance: a review", in Wilmott magazine, This paper* is a general, high-level description of our approach.
3) *"Mesh-free error integration in arbitrary dimensions: A numerical study of discrepancy functions", Computer Methods in Applied Mechanics and Engineering.* This paper focuses on error analysis for Reproducing Kernel Hilbert Space methods.
4) *« A new method for solving Kolmogorov equations in mathematical finance", Comptes Rendus Mathematique, Volume 355, Issue 6, June 2017, Pages 680-686.* This paper explains the Partial Differential Equation strategy for mathematical finance and provides numerical examples.
5) "Revisiting the method of characteristics via a convex hull algorithm" Journal of Computational Physics, October 2015 https://doi.org/10.1016/j.jcp.2015.05.043 applies this method for conservation laws.
6) A high dimensional framework for financial instruments valuation, 2013 an early attempt to describe multidimensional PDE for mathematical finance.
7) Optimally transported schemes 2008. treat the one dimensional case.

Algorithmic / numerical

We provide a library, named CodPy, which stands for "Curse of dimensionality in Python". It provides tools for machine learning, statistical learning, numerical simulations, and is based on our understanding of RKHS methods.

1) Codpy Tutorial is a gentle introduction to this library, focusing on machine learning.
2) *CodPy - Advanced Tutorial*, is a technical description of this library.
3) A kernel based reordering algorithm describes a central reordering algorithm for our application.
4) A kernel based polar factorization and the sampling algorithm with CodPy. In preparation. describes an algorithm to compute polar factorization. This algorithm is illustrated with a very handy tool, allowing to produce iid samples from any input distribution.
5) A kernel based algorithm to compute conditional expectations. In preparation. is a quite important algorithm for finance applications. We benchmark an implementation of this algorithm, using our framework CodPy, against a classical neural network one.
6) Hedging Strategies for Net Interest Income and Economic Values of Equity describes a prototype using CodPy, aiming to build sophisticated strategies for ALM purposes.
7) Kernel methods for stress test and reverse stress test is a support vector machine approach to these class problems, using CodPy.
8) Numerical results using codefi collects a number of academical tests for pricing purposes with our approach