The Adoption of Blockchain-based Decentralized Exchanges

Agostino Capponi

Columbia University

IAQF/Thalesians Seminars

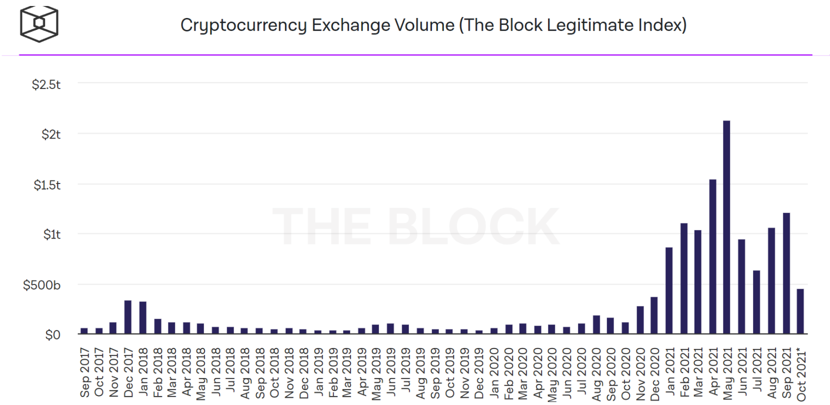
joint work with Ruizhe Jia

Outline

Background

- Blockchain technology has been disruptive:
  - Thousands of crypto tokens have been created.
  - Intermediaries for crypto trading have emerged
    - Crypto markets recorded **$1.17** trillion in exchange volumes in March 2021.
    - NYSE had a total volume of **$3.325** trillions in March 2021
  - Total capitalization of cryptocurrencies exceeded 2 trillions in early 2021

Crypto Trading Volume



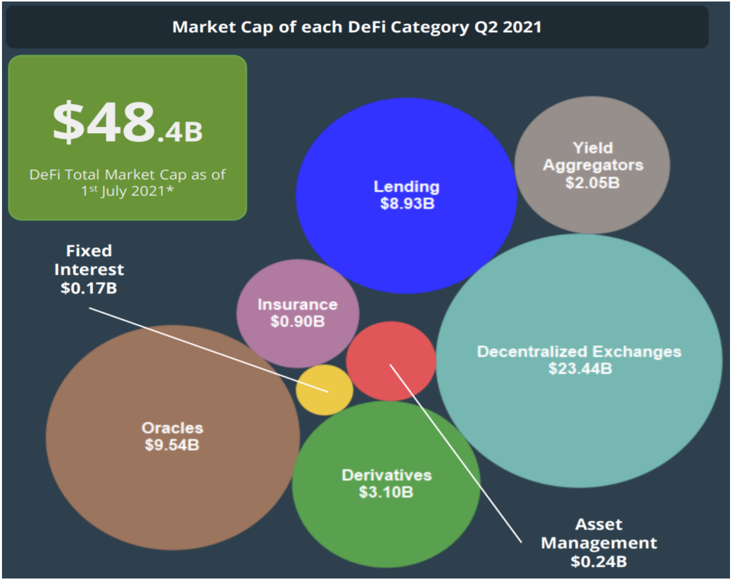
Centralized Intermediaries

- Majority of crypto transactions currently go through centralized exchanges operating like limit order books
- Most centralized exchanges are **unregulated**
- Risk of thefts and exit scams.
  - Mt.Gox, responsible for more than **70%** of bitcoin trading, suddenly closed its platform in early 2014
  - Other centralized exchanges subject to thefts and exit scams include Binance, BitKRX, BitMarket, PonziCoin

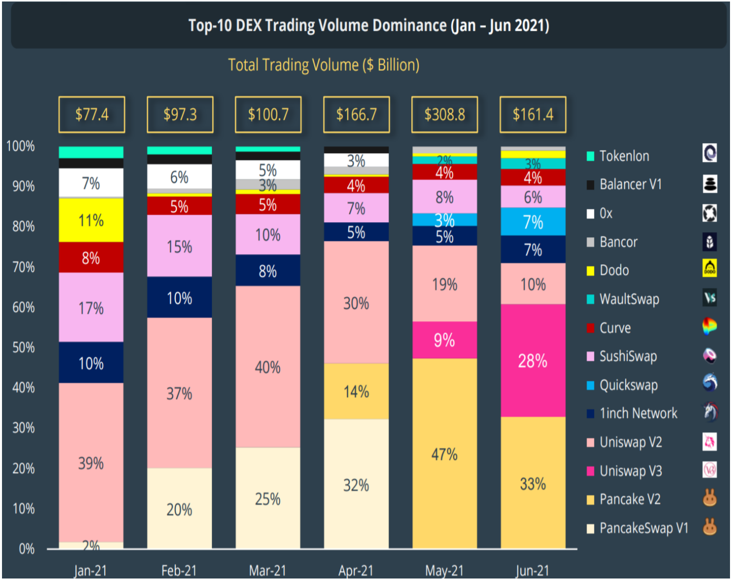
The Rise of DeFi

- In the mid of 2020, a new type of blockchain-based financial service emerged: **decentralized finance** (DeFi).
  - DeFi utilizes open-source smart contracts on blockchains
  - Provide financial services such as lending, borrowing, and trading without the involvement of a traditional financial intermediary

DeFi Ecosystem

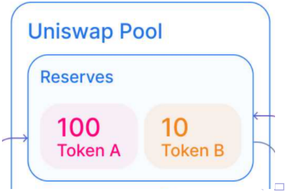


Main Decentralized Exchanges



Decentralized Echanges

- Basic Idea: **Pool liquidity**
  - Liquidity Providers:
    - Deposit tokens on both sides (e.g. USDT and ETH) to a liquidity pool
    - Share gains and losses
  - Liquidity Demanders:
    - Direct exchange of one crypto token for the other
    - Alternative to first selling token A for fiat currency, and then purchasing token B



Liquidity Provision

- Anyone who owns **both** tokens A and B can deposit.
- In return, receives pool tokens which prove his share of the AMM.
- Deposit at the current exchange rate in the pool
- Example: price of A = \$1; price of B = \$10.

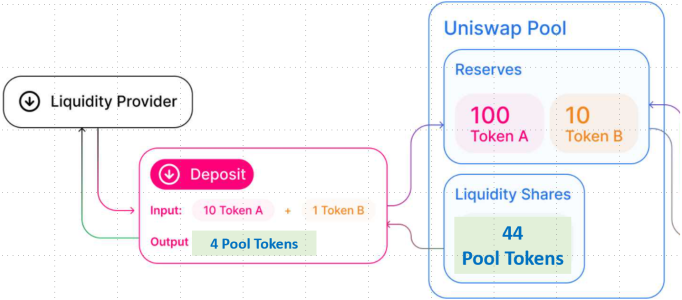


Figure: Source: Uniswap

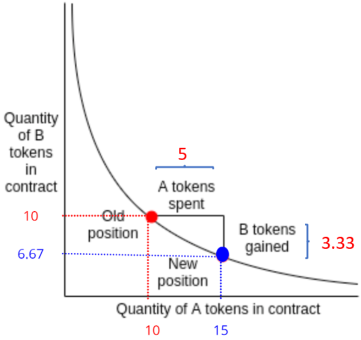
Automated Market Maker (AMM)

Pricing Mechanism:

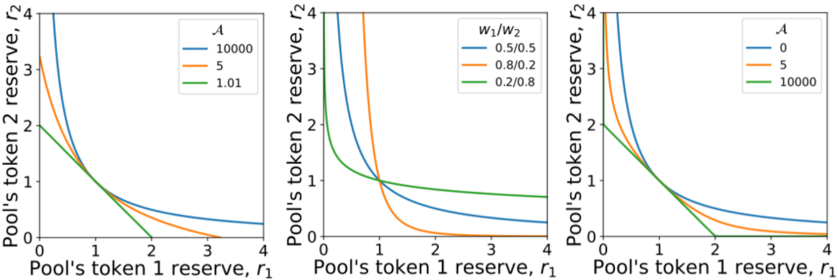
- A = contract balance of A token
- B = contract balance of B token
- $k = F(A, B)$ is the invariance factor

Example: $F(A, B) = A \times B$

- swap $\Delta_A = 5$ of A token for Δ_B of B token.
- $k = 10 \times 10 = (10 + 5) \times (10 - \Delta_B)$
- $\Delta_B = 3.33$



Convexity of Pricing Curves



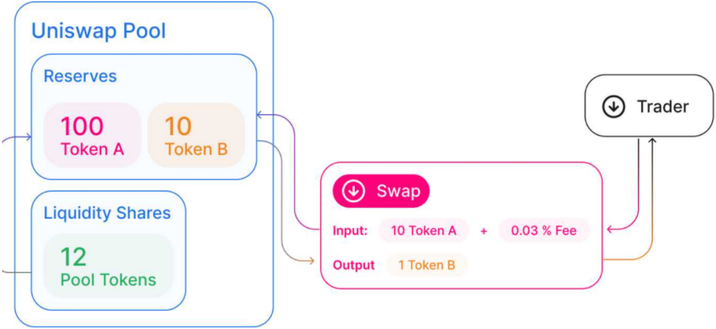
(a) Uniswap V2 & 3 (b) Balancer (c) Curve

Uniswap V2	Uniswap V3	Balancer	Curve
$r_1 \cdot r_2$	$\left(r_1 + \frac{\mathcal{C}_1}{\sqrt{\mathcal{A}} - 1}\right) \cdot \left(r_2 + \frac{\mathcal{C}_2}{\sqrt{\mathcal{A}} - 1}\right) = \frac{\mathcal{A} \cdot \mathcal{C}_1 \cdot \mathcal{C}_2}{(\sqrt{\mathcal{A}} - 1)^2}$	$\mathcal{C} = \prod_k r_k^{w_k}$	$\mathcal{A}\left(\frac{\sum_k r_k}{\mathcal{C}} - 1\right) = \frac{\left(\frac{\mathcal{C}}{n}\right)^n}{\prod_k r_k} - 1$

Figure: Source: Xu et al. (2021)

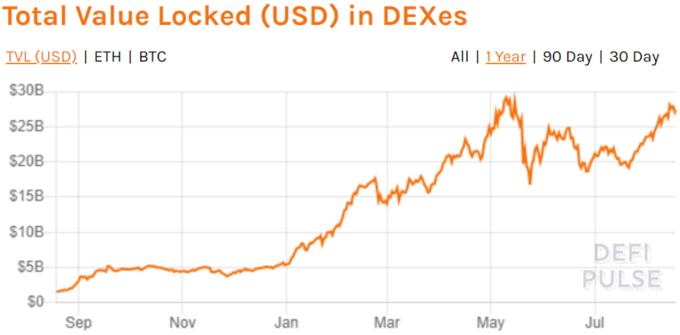
Trading Fee

- Investors pay a trading fee $f\Delta_A$ proportional to tokens A exchanged ($f \approx 0.3\%$).
- Most of trading fee added to the liquidity pool.
- The trading fee **incentivizes liquidity providers** to deposit.

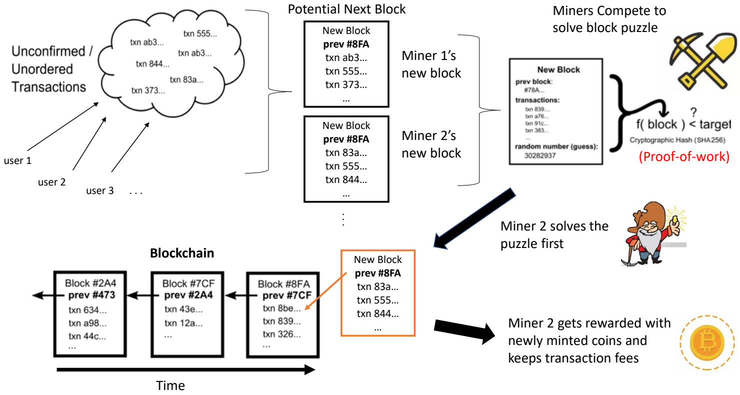


Are Liquidity Pools Material?

- Liquidity pools are **deep**: more than 360MUSD in USDC - ETH Uniswap pool alone which earned 730 kUSD fees on August 18



Execution Fees



- Orders executed first are not those arriving earlier, but those "bribing" miners with higher gas fees

Literature Review

- Scarce, but rapidly growing, literature on DeFi
  - Park (2021): front-running arbitrage
  - Lehar and Parlour (2021): comparison between centralized and decentralized echanges.
- Blockchain, Cryptocurrencies, and Tokenization:
  - Easley et al. (2019), Yermack (2017), Campbell (2016), Abadi and Brunnermeier (2018), Cong et al. (2020), Sockin and Xiong (2020), Biais et al. (2019), Reppen et al. (2019), Dai et al. (2021), Capponi et al. (2021), Prat and Walter (2018)

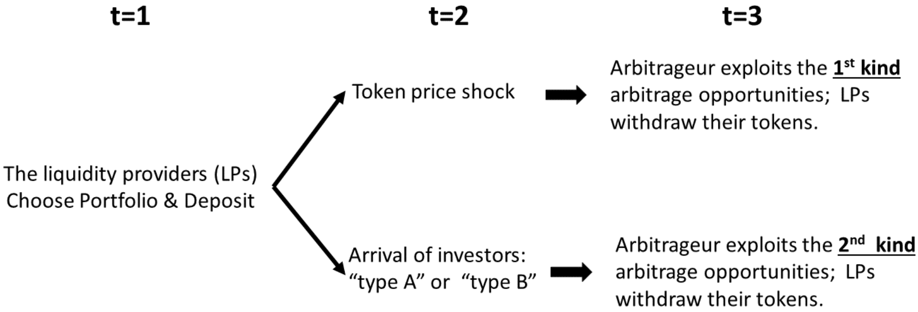
Key Research Questions

- Can the AMM incentivize liquidity providers to deposit?
- How does the design of the AMM affect trading activities and economic incentives of market participants?

Baseline Model Setup

- 3 periods indexed by t, $t = 1, 2, 3$.
- 3 kinds of agents: liquidity providers, an arbitrageur, and investors.
- 2 token types, A and B, are traded.
    - Token prices, $p_A^{(t)}$ and $p_B^{(t)}$ are **public information**

Model Timeline



AMM Pricing Function

- Smart contract utilizes a **twice continuously differentiable, convex pricing function** $F(x, y) : \mathbb{R}^2 \to \mathbb{R}$ to decide the exchange rate:

$$F(y_A + \Delta_A, y_B - \Delta_B) = F(y_A, y_B), 0 \leq \Delta_B \leq y_B$$

- Implicit Function Theorem on the above curve yields the **marginal exchange rate**

$$\lim_{\Delta_A \to 0} \frac{\Delta_B}{\Delta_A} = \frac{F_x}{F_y}\bigg|_{(x,y)=(y_A,y_B)}$$

Properties of Pricing Function

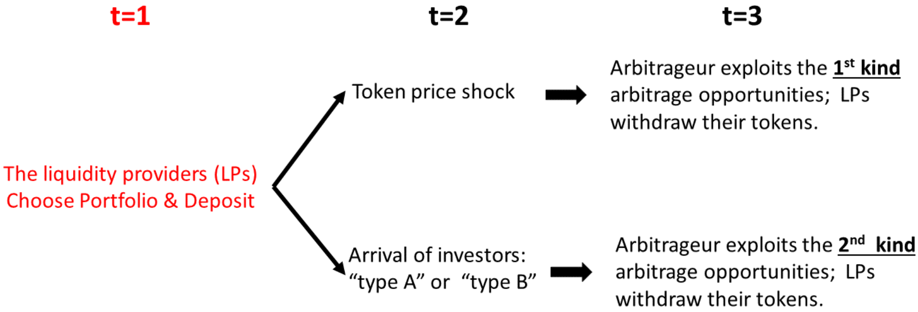
Assumption

$F(x, y) : \mathbb{R}^2 \to \mathbb{R}$ *satisfies the following properties:*

1. $F_x > 0, F_y > 0.$
2. $F_{xx} < 0, F_{yy} < 0, F_{xy} > 0.$
3. $\forall c \geq 0, c^l F(x, y) = F(cx, cy)$ *for some* $l > 0.$
4. $\lim_{x \to 0} \frac{F_x}{F_y} = \infty, \lim_{x \to \infty} \frac{F_x}{F_y} = 0, \lim_{y \to 0} \frac{F_x}{F_y} = 0, \lim_{y \to \infty} \frac{F_x}{F_y} = \infty.$

- Exchange rate is positive;
- Higher demand leads to higher price;
- Exchange rate invariant to scaling of deposited tokens;
- Supports trading for any token exchange rate in $[0, \infty)$.

Model Timeline



Liquidity Providers

- $n > 1$ liquidity providers, each provider i is endowed with a positive amount of A and B tokens, e_{A_i} and e_{B_i}
- At $t = 1$, each liquidity provider i maximizes its expected payoff at end of period 3

$$\max_{(y_{A_i}^{(1)}, y_{B_i}^{(1)}, x_{A_i}^{(1)}, x_{B_i}^{(1)})} \quad w_i \mathbb{E}[p_A^{(3)} y_A^{(3)} + p_B^{(3)} y_B^{(3)}] + \mathbb{E}[p_A^{(3)} x_{A_i}^{(1)} + p_B^{(3)} x_{B_i}^{(1)}]$$

$$\text{s.t.} \quad \left.\frac{F_x}{F_y}\right|_{(x,y)=(y_A^{(1)}, y_B^{(1)})} = \frac{p_A^{(1)}}{p_B^{(1)}}$$

$$y_{A_i}^{(1)} + x_{A_i}^{(1)} = e_{A_i}, y_{B_i}^{(1)} + x_{B_i}^{(1)} = e_{B_i}$$

$$y_{A_i}^{(1)}, y_{B_i}^{(1)}, x_{A_i}^{(1)}, x_{B_i}^{(1)} \geq 0.$$

Model Timeline

t=1

The liquidity providers (LPs) Choose Portfolio & Deposit

t=2

Token price shock

Arrival of investors: “type A” or “type B”

t=3

Arbitrageur exploits the **1st kind** arbitrage opportunities; LPs withdraw their tokens.

Arbitrageur exploits the **2nd kind** arbitrage opportunities; LPs withdraw their tokens.

Token Price Shock

- At $t = 2$, price changes of A and B tokens are caused by independent, idiosyncratic shocks ζ_A, ζ_B:

$$\zeta_A \sim Bern(\kappa_A), \zeta_B \sim Bern(\kappa_B), \zeta_A \perp \zeta_B, \kappa_A > \kappa_B,$$
$$p_i^{(2)} = p_i^{(1)} + \boldsymbol{\beta \zeta_i p_i}^{(1)}, i = A, B.$$

Investors' Arrival

- With probability $\frac{\kappa_I}{2}$, a "type A" investor arrives at $t = 2$
- A "type A" investor extracts private benefit of $(1+\alpha)p_A^{(1)}$ from using one A token on its corresponding platform.
- "Type A" solves:

$$\max_{\Delta Q_A^{(2)}, \Delta Q_B^{(2)}} \quad (1+\alpha)p_A^{(1)}\Delta Q_A^{(2)} + (1+f)p_B^{(1)}\Delta Q_B^{(2)}$$

$$\text{s.t.} \quad F(y_A^{(1)}, y_B^{(1)}) = F(y_A^{(1)} - \Delta Q_A^{(2)}, y_B^{(1)} - \Delta Q_B^{(2)})$$

$$y_A^{(1)} \geq \Delta Q_A^{(2)} \geq 0, \Delta Q_B^{(2)} \leq 0.$$

Model Timeline

t=1

The liquidity providers (LPs) Choose Portfolio & Deposit

t=2

Token price shock

Arrival of investors: "type A" or "type B"

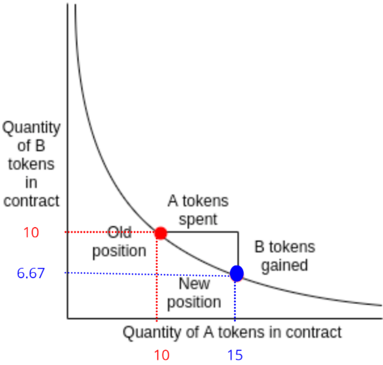
t=3

Arbitrageur exploits the **1st kind** arbitrage opportunities; LPs withdraw their tokens.

Arbitrageur exploits the **2nd kind** arbitrage opportunities; LPs withdraw their tokens.

Arbitrage Opportunities

- **1st kind:** If an idiosyncratic token price shock occurs, fair token exchange rate changes. *However*, the spot exchange rate in the AMM remains unchanged
- **2nd kind:** After an investor's trade, the ratio between the amount of A and B tokens deviates from $\frac{y_A^{(1)}}{y_B^{(1)}}$. *Reverse trade* may be profitable for a new investor



Arbitrageur

- One risk-neutral, deep-pocketed arbitrageur, submits an arbitrage order at $t = 3$ and attaches a gas fee $g_{arb}^{(3)}$ to its order.
  - If shock hits only B token, or a "type A" investor trades, then the arbitrageur profits from deviation of spot rate versus fair value exchange rate at AMM:

$$\max_{\Delta q_A^{(3)},\Delta q_B^{(3)}} \quad p_A^{(2)}(1+f)\Delta q_A^{(3)} + p_B^{(2)}\Delta q_B^{(3)}$$

$$\text{s.t.} \quad F(y_A^{(2)}, y_B^{(2)}) = F(y_A^{(2)} - \Delta q_A^{(3)}, y_B^{(2)} - \Delta q_B^{(3)})$$

$$\Delta q_A^{(3)} \leq 0, y_B^{(2)} \geq \Delta q_B^{(3)} \geq 0.$$

Assumption

The arbitrageur attaches a gas fee $g_{arb}^{(3)}$ equal to the highest possible profit from an arbitrage order (Glosten and Milgrom (1985)).

Gas Fee Paid by Arbitrageur

The gas fee paid for an arbitrage valued at around 40,000 is more than 38,000!

Transaction Action:	▸ Swap 16.290806524344807055 Ether For 19,106.558477 USDC On Balancer ▸ Swap 19,106.558477 USDC For 6,178.07283593 sil On Sushiswap ▸ Swap 6,165.71669026 sil For 32.98093614065103254 Ether On Sushiswap
Value:	0 Ether ($0.00)
Transaction Fee:	12.006066915817169675 Ether ($38,161.04)

Existence and Uniqueness of Equilibrium

Proposition

For any $F(x, y)$, $\alpha, \beta, f, \kappa_I, \kappa_A, \kappa_B$, there exists a unique subgame perfect Nash equilibrium.

Proof Sketch:

- We follow the proof strategy of Zermelo's theorem.
- To establish existence, we identify a strategy profile through backward induction;
- Multiple equilibria are never encountered in the process of backward induction. Hence, the identified strategy profile is the unique subgame perfect equilibrium.

When Does the Arbitrageur Profit?

Proposition

If $\beta > f$, the arbitrageur earns a positive profit $\pi(y_A^{(2)}, y_B^{(2)}, p_B^{(2)}, p_A^{(2)})$ from the optimal arbitrage trade. Moreover, such profit and the unique optimal trading size $|\Delta q_A^{(3)*}|$ for A tokens and $|\Delta q_B^{(3)*}|$ for B tokens are increasing in β and decreasing in f.

Competition between Arbitrageurs and Liquidity Providers

Proposition

If $\beta > f$ and the token price shock hits in period 2, then an optimal arbitrage order is the first order executed in period 3. The arbitrage yields a loss $w_i \pi(y_A^{(2)}, y_B^{(2)}, p_B^{(2)}, p_A^{(2)})$ for liquidity provider i.

- For the arbitrageur, the first execution has value $\pi(y_A^{(2)}, y_B^{(2)}, p_B^{(2)}, p_A^{(2)})$.
- For liquidity provider i, the first execution has a value $w_i \pi(y_A^{(2)}, y_B^{(2)}, p_B^{(2)}, p_A^{(2)}), w_i < 1$.
- For liquidity providers, the **arbitrage problem is unavoidable**
- The second execution is worth 0
- Liquidity providers attach zero gas fee to their exit orders.

Optimal Investors' Trades

Lemma

If $f < \alpha$, the arriving investor trades and earns a positive surplus from the transaction. Maximized surplus and optimal trading quantities $\Delta Q_A^{(2,*)}$ and $\Delta Q_B^{(2,*)}$ are increasing in α, and decreasing in f.

Adoption of AMM

Key tradeoff: fees revenue from investors' trading vs **arbitrage loss**.

Theorem

A "liquidity freeze", i.e., $(y_{A_i}^{(1)}, y_{B_i}^{(1)}) = (0, 0)$ *for any* $i = 1, \ldots, n$*, occurs if and only if the token exchange rate is sufficiently volatile, i.e.,* $\beta \geq \overline{\beta_{frz}}$*, where* $\overline{\beta_{frz}} \in [0, +\infty]$*. Moreover, the threshold* $\overline{\beta_{frz}}$ *is increasing in* $\alpha, \kappa_I, \kappa_B$*, and decreasing in* κ_A*.*

Negative Externalities

Proposition

Suppose that "liquidity freeze" is not an equilibrium outcome. The expectation and variance of the gas fee, $\mathbb{E}[g_{arb}^{(3)}]$ *and* $Var[g_{arb}^{(3)}]$*, are both increasing in* β *and in the amount of token* $y_A^{(1)}$ *and* $y_B^{(1)}$ *deposited by liquidity providers.*

- High transaction fees may present barriers to entry for new platforms

Empirical Implications

Proposition

The incentive to deposit, i.e., $\mathbb{E}[R_D] - \mathbb{E}[R_{ND}]$*, is increasing in* α, κ_I *and decreasing in* β*.*

- An increase in token exchange rate volatility ($\beta \uparrow$) decreases the amount of token deposited at the AMM.
- An increase in trading volume increases ($\alpha, \kappa_I \uparrow$) the amount of token deposited at the AMM.

Proposition

The expectation and variance of the gas fee, $\mathbb{E}[g_{arb}^{(3)}]$ *and* $Var[g_{arb}^{(3)}]$*, are increasing in* β*.*

- Higher average and volatility of the gas fee for more volatile pairs ($\beta \uparrow$)

Data

- The dataset contains histories of all trades, deposits, and withdrawals for a sample of 80 AMMs with actively traded pairs.
  - Among the 80 AMMs, 40 of them are from Uniswap, and the rest are from Sushiswap.
  - 7 pairs consist of only stable coins pegged to one US dollar, and they are denoted as **“stable pairs”**.

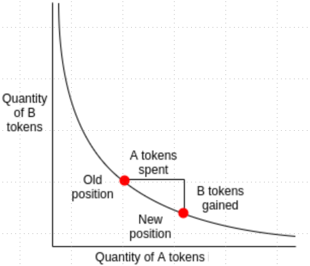
Regression Results

	Dependent variable: Deposit Inflow Rate		
	(Regr.1)	(Regr.2)	(Regr.3)
Intercept	0.023	−0.103	−0.018
	(0.077)	(0.073)	(0.070)
Exchange Rate Volatility	−0.394**		−1.451***
	(0.182)		(0.405)
Trading Volume		0.039***	0.052***
		(0.015)	(0.020)
Week fixed effects?	yes	yes	yes
AMM fixed effects?	yes	yes	yes
Observations	750	750	750
R^2	0.11	0.14	0.17
Note:			*p<0.1; **p<0.05; ***p<0.01

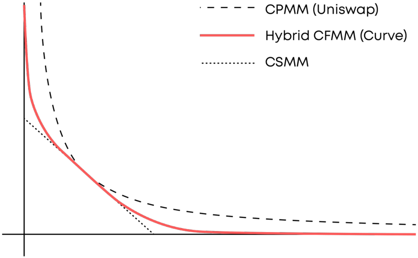
Regression Results

	Dependent variables:	
	Gas Price Volatility (Regr.1)	Gas Price (Regr.2)
Intercept	189.48***	137.34***
	(16.13)	(1.61)
Stable	-64.34***	-10.51***
	(20.02)	(2.78)
Week fixed effects?	yes	no
Day fixed effects?	no	yes
Exchange fixed effects?	yes	yes
Observations	750	4,161,126
R^2	0.24	0.04
Note:	*p<0.1; **p<0.05; ***p<0.01	

Design of the AMM



(a) Pricing Curve



(b) Different Curvatures

Slope of the Curve = negative of Spot Exchange Rate: $-\frac{F_x}{F_y}$

Larger curvature $\rightarrow$ Larger slippage $\rightarrow$ More expensive to trade

The Curvature of Pricing Curve

- Consider the following family of pricing functions:

$$F_k(x,y) = (1-k)\ A\ F_0(x,y) + k\ F_1(x,y),$$

where $F_0(x,y) = p_A^{(1)}x + p_B^{(1)}y$, $F_1(x,y) = xy$, $k \in [0,1]$, and $A = \left(\frac{y_A^{(1)}y_B^{(1)}}{p_A^{(1)}p_B^{(1)}}\right)^{1/2}$ is a scaling coefficient.

- The curvature of the pricing curve $F_k(x,y) = C$ is increasing in k.

Optimal Curvature

- **Small Curvature**: Arbitrage problem is too severe, and the AMM sells to the investor too cheaply.
- **Large Curvature**: Investor does not trade much, thus little fee revenue, but arbitrage problem is less severe

Proposition

There exists a threshold $k^* \in (0,1)$ such that, at equilibrium, the expected payoff of liquidity providers is increasing in k, for $k \in [0, k^*]$, and decreasing in k, for $k \in [k^*, 1]$.

If there is a "liquidity freeze" at $k = k^*$, then there is a "liquidity freeze" for any other $k \in [0,1]$.

The **socially optimal** *pricing curve, i.e., under which the sum of agents' equilibrium expected payoffs is maximized, is attained at k^*.*

Does Pooling More Tokens Reduce the Arbitrage Problem?

- The answer is NO.
- **Misconception:** pooling more tokens may alleviate the arbitrage problem due to diversification effects.
- Three tokens: A, B, and C.
- Two AMMs: one handles A and B tokens only, while the other handles all three tokens.
- Both AMMs utilize a constant product function, that is, $F_{AB}(x, y) = xy$ for the first AMM, and $F_{ABC}(x, y, z) = xyz$ for the second AMM.

Pooling More Tokens Exacerbates the Arbitrage Problem

Proposition

The expected arbitrage loss ratio of the AMM pooling A and B tokens, $\mathbb{E}\left[\frac{\pi_{AB}(y_A^{(2)}, y_B^{(2)}, p_B^{(2)}, p_A^{(2)})}{p_A^{(1)} y_A^{(1)} + p_B^{(1)} y_B^{(1)}}\right]$, *is smaller than the expected arbitrage loss ratio of the AMM which pools A, B, and C tokens,* $\mathbb{E}\left[\frac{\pi_{ABC}(y_A^{(2)}, y_B^{(2)}, y_C^{(2)}, p_C^{(2)}, p_B^{(2)}, p_A^{(2)})}{p_A^{(1)} y_A^{(1)} + p_B^{(1)} y_B^{(1)} + p_C^{(1)} y_C^{(1)}}\right]$.

- Arbitrage problem becomes even more severe.
- Arbitrage becomes more likely as number of tokens in the AMM increases.
- For each realized arbitrage opportunity, the arbitrageur can extract a larger portion of the shocked token from the AMM deposit.

Conclusion

- New framework for analyzing and designing decentralized exchanges
- Adoption by liquidity providers may not occur if their revenues from trading fees are lower than losses imposed by arbitrageurs
- “Unstable pairs” and adoption of AMM impose large negative externalities on the underlying Ethereum blockchain.
- Curvature of pricing curve governs trade-offs between severity of arbitrage problem and investors’ willingness to trade

Thank You!

Abadi, J. and Brunnermeier, M. (2018), Blockchain Economics, NBER Working Papers 25407, National Bureau of Economic Research, Inc.

Biais, B., Bisière, C., Bouvard, M. and Casamatta, C. (2019), 'The Blockchain Folk Theorem', *The Review of Financial Studies* **32**(5), 1662–1715.

Campbell, R. H. (2016), Cryptofinance, Working paper.

Capponi, A., Olafasson, S. and Alsabah, H. (2021), 'Proof-of-Work Cryptocurrencies: Does Mining Technology Undermine Decentralization?', *Major Revision at Management Science* .

Cong, L. W., Li, Y. and Wang, N. (2020), Token-Based Platform Finance, NBER Working Papers 27810, National Bureau of Economic Research, Inc.

Dai, M., Jiang, w., Kou, S. and Qin, C. (2021), 'Bitcoin Mining and Electricity Consumption', *Working Paper* .

Easley, D., Maureen, O. and Basu, S. (2019), 'From mining to markets: The evolution of bitcoin transaction fees', *Journal of Financial Economics* **134**(1), 91–109.

Glosten, L. R. (1994), 'Is the electronic open limit order book inevitable?', *The Journal of Finance* **49**(4), 1127–1161.

Glosten, L. R. and Milgrom, P. R. (1985), 'Bid, ask and transaction prices in a specialist market with heterogeneously informed traders', *Journal of Financial Economics* **14**(1), 71–100.

Lehar, A. and Parlour, C. (2021), Decentralized exchanges, Working paper.

Park, A. (2021), The Conceptual Flaws of Constant Product Automated Market Making, Working paper.

Prat, J. and Walter, B. (2018), An Equilibrium Model of the Market for Bitcoin Mining, CESifo Working Paper Series 6865, CESifo.
URL: *https://ideas.repec.org/p/ces/ceswps/*$_6$865*.html*

Reppen, M., Sircar, R. and Zongxi, L. (2019), 'A Mean Field Games Model for Cryptocurrency Mining'.

Sockin, M. and Xiong, W. (2020), A model of cryptocurrencies, Working Paper 26816, National Bureau of Economic Research.

Xu, J., Vavryk, N., Paruch, K. and Cousaert, S. (2021), 'Sok: Decentralized exchanges (dex) with automated market maker (AMM) protocols'.

Yermack, D. (2017), 'Corporate Governance and Blockchains', *Review of Finance* **21**(1), 7–31.

Benefits of DEX

- **No counterparty risk:**
  - Settlement of transactions is instantaneous, after they are confirmed and included on the blockchain.
  - Prior to settlement, traders retain full control of their tokens.
- **Financial inclusion:**
  - Any token holder can become a liquidity provider by depositing their tokens and earning fees from trading activities.

Existence and Uniqueness of Equilibrium

Proposition

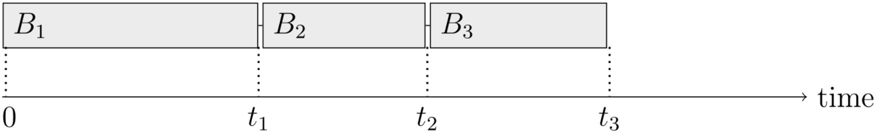
For any $F(x, y)$, $\alpha, \beta, f, \kappa_I, \kappa_A, \kappa_B$, there exists a unique subgame perfect Nash equilibrium.

Proof Sketch:

- We follow the proof strategy of Zermelo's theorem.
- To establish existence, we identify a strategy profile through backward induction;
- Multiple equilibria are never encountered in the process of backward induction. Hence, the identified strategy profile is the unique subgame perfect equilibrium.

A Continuous Time Extension

Continuous time dynamic game between LPs and arbitrageurs:



- Time required for mining a new block follows exponential distribution: $t_1, t_2, \ldots$ are the times a new block is mined.
- Dynamics of fundamental token values, $p_j(t)$, $j = A, B$:

$$dp_j(t) = \mu(t, p_j(t))dt + \sigma(t, p_j(t))dW_t^{(j)}$$

- Arbitrage opportunity exists at stopping time τ, $t_{k-1} \leq \tau < t_k$, if $p_A(\tau)/p_B(\tau) > \delta_1$, or $p_A(\tau)/p_B(\tau) < \delta_2$. δ_1, δ_2 determined by token exchange rate recorded in the previous block at t_{k-1}.
- At τ, all arbitrageurs enter into a dynamic, ascending, first-price auction, with deadline t_k.

A Continuous Time Extension

Challenges:

- Token prices, $p_A(t), p_B(t)$ change while arbitrageurs bid. Do they need to dynamically adjust the order?
- Arbitrage opportunity may even disappear before the next block is generated, i.e. , $\delta_2 < p_A(t_k)/p_B(t_k) < \delta_1$.
- The random auction deadline adds another layer of complexity.
- Other components:
  - Entering auction has a small cost
  - Network delay makes other arbitrageurs' bids not observable. Blind Raising?

Proof Sketch

Constraint is an implicit curve, where F is unspecified. Feasible set is **non-convex**.

- Using Implicit function theorem, rewrite the constraint as

$$\Delta q_B^{(3)} = g(\Delta q_A^{(3)}), -\infty < \Delta q_A^{(3)} \leq 0,$$

  where g is a twice differentiable concave function.
- Obtain an equivalent single-variable, **convex** optimization problem:

$$\max_{-\infty < \Delta q_A^{(3)} \leq 0} \quad p_A^{(1)}(1+f)\Delta q_A^{(3)} + p_B^{(1)}(1+\beta)g(\Delta q_A^{(3)})$$

- Derivative of above objective function:

$$p_A^{(1)}(1+f) - (-p_B^{(1)}(1+\beta)\frac{dg}{d(\Delta q_A^{(3)})})$$

- If $1+f > 1+\beta$, then $\Delta q_A^{(3)*} = \Delta q_B^{(3)*} = 0$; if $1+f < 1+\beta$, then at $\Delta q_A^{(3)*}$, $p_A^{(1)}(1+f) = (-p_B^{(1)}(1+\beta)\frac{dg}{d(\Delta q_A^{(3)})})$.

Main Research Questions

- Can the AMM provide sufficient incentives for provision of liquidity?
- Will there be any market breakdown where the liquidity reserve of the AMM is drained?
- What kind of tokens are this new type of exchanges suitable for?
- How does the convexity of pricing function affect trading activities?

Variables

- **Trading Volume (of Investors)**:

$$Volume_{jt} = \left(\frac{TradeA_{jt}}{TokenA_{jt}} \times \frac{TradeB_{jt}}{TokenB_{jt}} \right)^{1/2},$$

  - $TradeA_{jt}$, $TradeB_{jt}$ are the total token A and token B traded by the investors with swap orders at AMM j in week t.

- **Gas Price Volatility:** standard deviation of gas price attached to all transactions executed on AMM j in week t.

Economic Significance

- A one-standard-deviation increase in weekly spot rate volatility decreases the deposit flow rate by 25% standard deviations.
- A one-standard-deviation increase in trading volume increases deposit flow rate by 35% standard deviations.

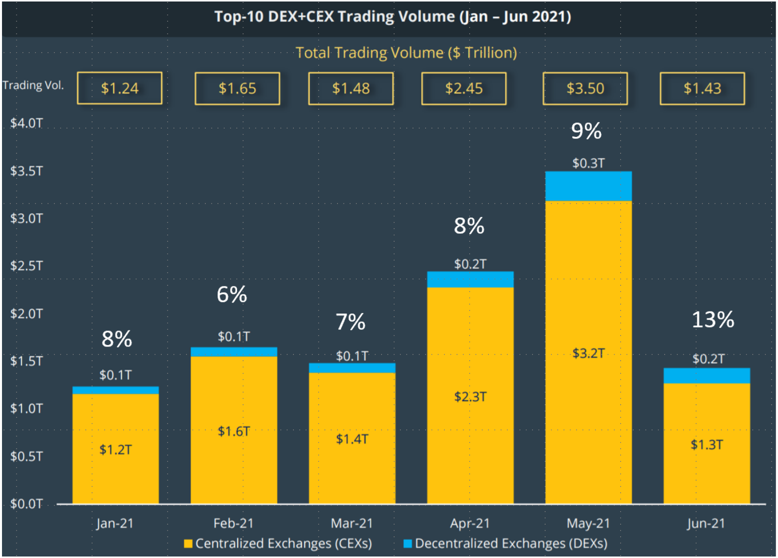
Variables

- **Token Exchange Rate Volatility**: standard deviation of log spot rate between two tokens deposited in the AMM, in each week.
- **Deposit Flow Rate**:

$$Depositflow_{jt} = sgn(DepositA_{jt}) \times \left(\frac{DepositA_{jt}}{TokenA_{jt}} \times \frac{DepositB_{jt}}{TokenB_{jt}} \right)^{1/2}$$

- $DepositA_{jt}$, $DepositB_{jt}$ are the total tokens A and token B by liquidity providers of AMM j during week t
- $TokenA_{jt}$, $TokenB_{jt}$ are the initial deposits in AMM j during week t.

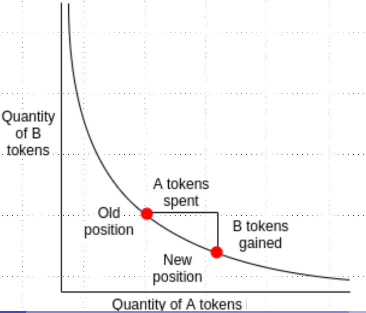
Total Intermediated Trading Volume



Automated Market Maker

A new form of market making —**Automated Market Maker** (AMM):

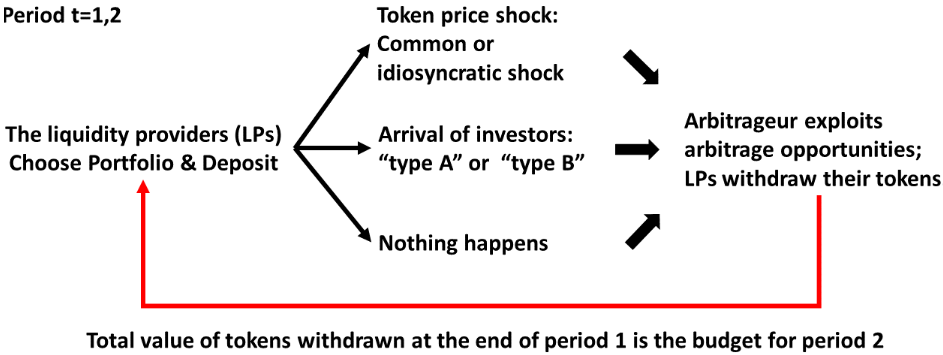
- Exchange amount Δ_A of token A for amount Δ_B of token B.
- **Uniswap's Constant Product Function:** Δ_A and Δ_B must satisfy $(100 + \Delta_A)(10 - \Delta_B) = 100 * 10 = 1000$.
- **General Pricing Function:** $F(100 + \Delta_A, 10 - \Delta_B) = F(100, 10)$, where $F(x, y)$ is an arbitrary pricing function.



Is Arbitrage Loss Impermanent?

- Does the token value loss still exist if the token exchange rate reverts back to its initial level in subsequent periods?
- Quick answer: NO.
- **Misconception:** if the token exchange rate is hit by a shock in the opposite direction, another arbitrage will occur and bring the ratio of deposits back to the initial ratio. Hence, there would be **no token value loss** from arbitrage.

New Timeline



Measurement of "Impermanent Loss"

Definition

The "impermanent loss" is defined as:

$$IL\left(\frac{p_A^{(0)}}{p_B^{(0)}}, \frac{p_A^{(2,3)}}{p_B^{(2,3)}}\right) := 1 - \frac{p_A^{(2,3)} x_2 + p_B^{(2,3)} y_2}{p_A^{(2,3)} x_1 + p_B^{(2,3)} y_1}, \tag{1}$$

where $x_1, y_1, x_2, y_2 > 0$ *are specified by the following constraints:*

$$F(x_1, y_1) = F(x_2, y_2), \frac{F_x(x_1, y_1)}{F_y(x_1, y_1)} = \frac{p_A^{(0)}}{p_B^{(0)}}, \frac{F_x(x_2, y_2)}{F_y(x_2, y_2)} = \frac{p_A^{(2,3)}}{p_B^{(2,3)}}.$$

- Intend to capture the magnitude of token value loss from depositing relative to not depositing.
- IL $= 0$ if the token price reverts, i.e., $\frac{p_A^{(0)}}{p_B^{(0)}} = \frac{p_A^{(2,3)}}{p_B^{(2,3)}}$.

Token Price Shock

CAN WE SIMPLIFY AND GET RID OFF THE COMOVEMENT PART IN THIS SLIDE? HOWEVER, YOU CAN COPY AND PAST THIS SLIDE IN THE APPENDIX OF THE PRESENTATION Token prices either **co-move** or **move independently**:

- With probability θ, token price changes driven by **common shock** ζ_{com}:

$$\zeta_{com} \sim Bern(\kappa_{com}), p_i^{(2)} = (1 + \beta\zeta_{com})p_i^{(1)}, i = A, B.$$

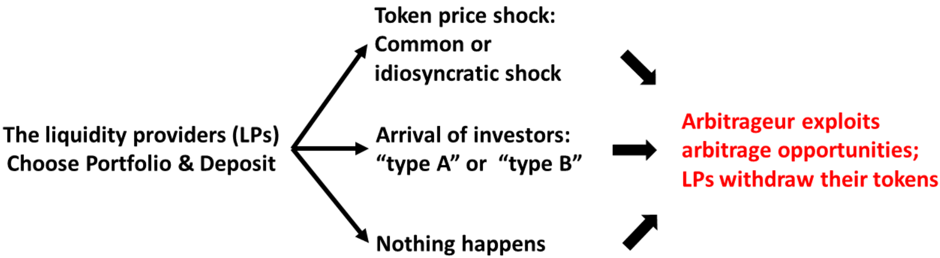
- With probability $1 - \theta$, token price changes driven by independent, **idiosyncratic shocks** ζ_A, ζ_B:

$$\zeta_A \sim Bern(\kappa_A), \zeta_B \sim Bern(\kappa_B), \zeta_A \perp \zeta_B, \kappa_A > \kappa_B,$$
$$p_i^{(2)} = (1 + \beta\zeta_i)p_i^{(1)}, i = A, B.$$

Result

- The gas price volatility for "stable pairs" is significantly lower than for "non-stable pairs": around 35% lower;
- The gas price for "stable pairs" is significantly lower than for "non-stable pairs": around 8% lower

Timeline


The liquidity providers (LPs)
Choose Portfolio & Deposit
Token price shock:
Common or
idiosyncratic shock
Arrival of investors:
"type A" or "type B"
Nothing happens
Arbitrageur exploits
arbitrage opportunities;
LPs withdraw their tokens

Is “Impermanent Loss” the right measurement?

- **Misconception:** after arbitrage loss occurs in period 1, if the token exchange rate is likely to revert to its initial level, then it is optimal for the liquidity providers to keep depositing in period 2 and wait for exchange rate reversion.
- **Our Result:** when the probability of token exchange rate reversion increases, the expected “impermanent loss” decreases, but the liquidity providers’ incentive to deposit becomes weaker.
- The optimal action is to hold the token that is likely to revert in the portfolio and not provide liquidity at the AMM.
- Reason: IL uses wrong benchmark and fails to fully account for the opportunity cost.

Examples

- Beginning of Period 1:
  - Price of A token = 1; Price of B token = 1;
  - Deposit: 100 A tokens, 100 B tokens.
- Period 1:
  - Price Shock: Price of A token = 4; Price of B token = 1;
  - Deposit after arbitrage: 50 A tokens, 200 B tokens;
  - Deposit Value =400; if not deposit, total value = 4*100+100 = 500
  - IL = 20%
- Period 2
  - Price Shock: Price of A token = 4; Price of B token = 4;
  - Deposit after arbitrage: 100 A tokens, 100 B tokens;
  - Deposit Value =800; IL = 0%
  - if not deposit in period 2, total value = 1000

Performance of tokens

	Q1 2021	Q2 2021	YTD (1st Jan - 1st July 2021)
Top-5 DeFi Tokens			
UNI	442%	-31%	273%
LINK	159%	-33%	74%
AAVE	332%	-34%	184%
CAKE	2,843%	-24%	2,140%
MKR	261%	24%	348%
Top-5 Cryptocurrencies			
BTC	103%	-40%	21%
ETH	159%	19%	209%
BNB	710%	0.3%	712%
ADA	555%	16%	661%
DOGE	1,055%	366%	5,289%
Top-5 DEX Tokens			
UNI	442%	-31%	273%
CAKE	2,843%	-24%	2,140%
RUNE	578%	-15%	479%
SUSHI	412%	-44%	185%
BNT	484%	-55%	165%
Top-5 NFT Tokens			
THETA	555%	-43%	272%
CHZ	2,242%	-49%	1,094%
ENJ	1,845%	-55%	784%
MANA	1,186%	-43%	633%
ECOMI	35,173%	-82%	6,332%
Top-5 Gaming Tokens			
ENJ	1,845%	-55%	784%
AXS	960%	0.5%	965%
UOS	367%	-30%	227%
ALICE	-30%	-73%	-81%
GALA	1,628%	-66%	483%

Comparison to LOB

- In a limit-order book market (e.g., Glosten and Milgrom (1985) and Glosten (1994)), market markers can be **adversely selected**
  - Mitigate adverse selection by charging a bid-ask spread
- In the AMM, the arbitrage exists even if there is **complete information.**
  - No bid-ask spread and impossibility to front-run the arbitrage order

Exchange Rate Volatility, Trading Volume, and Deposit

$$Depositflow_{jt} = \gamma_j + \gamma_t + \rho_1 Volatility_{jt} + \epsilon_{jt}$$

$$Depositflow_{jt} = \gamma_j + \gamma_t + \delta_1 Volume_{jt} + \epsilon_{jt}$$

$$Depositflow_{jt} = \gamma_j + \gamma_t + \rho_2 Volatility_{jt} + \delta_2 Volume_{jt} + \epsilon_{jt},$$

- **Model Prediction:** $\rho_1, \rho_2 < 0$, and $\delta_1, \delta_2 > 0$.

Competition on first execution

- Arbitrage opportunity can be exploited by the arbitrageur only if its order is confirmed before the liquidity providers exit.
- Both arbitrageur and liquidity providers **attach gas fees** to their orders.
- Orders included on the underlying blockchain and executed in decreasing order of gas fees, and any tie broken uniformly at random.

Token Exchange Rate Volatility and Gas Price

$$GasVolatility_{jt} = \gamma_t + \gamma_{Uniswap} + \kappa_A \mathbb{1}_{StablePair} + \epsilon_{jt}$$
$$Gas_{js} = \gamma_t + \gamma_{Uniswap} + \kappa_B \mathbb{1}_{StablePair} + \epsilon_{js}$$

- Segments: "stable pairs" and "unstable pairs"
- Coefficients κ_A, κ_B quantify the differences in gas fee and in gas price volatility between "stable pairs" and "unstable pairs", respectively.
- We expect that $\kappa_A, \kappa_B < 0$ from our theoretical analysis

Does Pooling More Tokens Reduce the Arbitrage Problem?

With probability θ, the price movements of A, B, and C tokens are driven by a common shock ζ_{com}:

$$\zeta_{com} \sim Bern(\kappa), p_i^{(2)} = (1 + \beta\zeta_{com})p_i^{(1)}, i = A, B, C. \quad (2)$$

With probability $1 - \theta$, they are determined by independent, idiosyncratic shocks $\zeta_A, \zeta_B, \zeta_C$, respectively:

$$\begin{aligned} &\zeta_i \sim Bern(\kappa), \zeta_A \perp \zeta_B, \perp \zeta_B \perp \zeta_C, \perp \zeta_A \perp \zeta_C, \\ &p_i^{(2)} = (1 + \beta\zeta_i)p_i^{(1)}, i = A, B, C. \end{aligned} \quad (3)$$

What Leads to Adoption

- When tokens become more attractive for investors (α increases) and the arrival rate of investors goes up (κ_I increases), the **expected trading volume** increases and thus liquidity providers collect a higher trading fee.
- When the tokens are more likely to be hit by a common shock (θ increases) and **co-move**, arbitrage opportunities are less likely to occur.
- When the token exchange rate is more **volatile** (magnitude of the price shock β and arrival rate of the shock κ_A both increase), the arbitrage becomes more costly for liquidity providers.
- When κ_B increases, the difference in expected return of the two tokens $(1+\beta)(\kappa_A - \kappa_B)$ decreases, and thus the **opportunity cost** of holding both tokens and providing liquidity decreases.

Implications

- For exchanges:
  - the curvature of the pricing curve used by the AMM governs the trade-offs between the severity of the arbitrage problem and the investors' willingness to trade. these two forces are well balanced, a

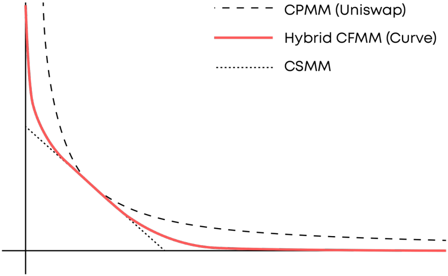
    "liquidity freeze" is least likely to occur, the deposit efficiency is the highest, and social welfare is maximized.
  - pooling more than two tokens in the same AMM does not alleviate the arbitrage problem.
- For token holders: **only deposit into AMMs whose token prices are stable and highly correlated, or whose trading volumes are high.**

Thank You!

Token Withdrawal

- In period 3, the liquidity provider i withdraws his tokens from the AMM by submitting an order and attaching a non-negative gas fee $g_{(lp,i)}^{(t)}$ to it.
- When the withdraw order is executed, the liquidity provider i pays the attached gas fee and receives w_i portion of the total A tokens in the liquidity pool and w_i portion of the total B tokens in the liquidity pool.

Different Pricing Curves



Total Market Capitalization

Exchange Rate Volatility, Trading Volume, and Deposit

$$Depositflow_{jt} = \gamma_j + \gamma_t + \rho_1 Volatility_{jt} + \epsilon_{jt} \tag{4}$$

$$Depositflow_{jt} = \gamma_j + \gamma_t + \delta_1 Volume_{jt} + \epsilon_{jt} \tag{5}$$

$$Depositflow_{jt} = \gamma_j + \gamma_t + \rho_2 Volatility_{jt} + \delta_2 Volume_{jt} + \epsilon_{jt}, \tag{6}$$

- The coefficients ρ_1, ρ_2 quantify the sensitivity of deposit flow on token volatility.
- The coefficients δ_1, δ_2 give the sensitivity of deposit flow on trading volume.
- Model Prediction: $\rho_1, \rho_2 < 0$, and $\delta_1, \delta_2 > 0$.
- We cluster our standard errors at the AMM level.

Token Exchange Rate Volatility and Gas Price

$$GasVolatility_{jt} = \gamma_t + \gamma_{Uniswap} + \kappa_A \mathbb{1}_{StablePair} + \epsilon_{jt} \quad (7)$$

$$Gas_{js} = \gamma_t + \gamma_{Uniswap} + \kappa_B \mathbb{1}_{StablePair} + \epsilon_{js} \quad (8)$$

- Segments: 'stable pairs" and "unstable pairs"
- Coefficients κ_A, κ_B quantify the differences in gas fee and in gas price volatility between "stable pairs" and "unstable pairs", respectively.
- We expect that $\kappa_A, \kappa_B < 0$ from our theoretical analysis

Data

- Identify orders:
  - If an investor trades in (takes out) both tokens in a transaction, then we identify the transaction as a deposit (withdrawal);
  - if instead, the investor trades in one token and takes out the other token, we identify the transaction as a swap.
  - We can calculate and track the total liquidity reserve of both tokens in AMMs and the spot rate of the exchange
- 12-week period from Dec 22, 2020 to March 20, 2021. The number of AMMs initiated by Dec 22, 2020 is only 49; 5 pairs are "stable pairs".

Descriptive Statistics

	N	Mean	SD	10th	50th	90th
		Panel A: Weekly-level Data				
Log Rate Volatility, All	588	0.062	0.053	0.006	0.050	0.126
Log Rate Volatility, Stable	60	0.005	0.004	0.003	0.003	0.010
Log Rate Volatility, Unstable	528	0.068	0.052	0.022	0.055	0.129
Token Inflow Rate, All	588	0.037	0.266	-0.190	0.007	0.248
Token Inflow Rate, Stable	60	0.116	0.389	-0.154	0.004	0.459
Token Inflow Rate, Unstable	528	0.028	0.246	-0.192	0.007	0.233
Trading Volume, All	588	2.318	2.291	0.356	1.608	5.123
Trading Volume, Stable	60	1.710	1.521	0.587	1.333	3.384
Trading Volume, Unstable	528	2.387	2.352	0.337	1.646	5.496
Gas Price Volatility, All	588	199.761	251.065	57.880	138.391	362.430
Gas Price Volatility, Stable	60	90.811	43.982	46.152	78.732	164.043
Gas Price Volatility, Unstable	528	212.142	261.677	60.819	145.488	387.538

Descriptive Statistics

	N	Mean	SD	10th	50th	90th
			Panel B: Transaction-level Data			
Gas Price (Gwei), All	2,859,992	154.287	322.632	57.000	120.500	244.880
Gas Price (Gwei), Stable	216,702	122.915	95.813	52.000	104.000	202.000
Gas Price (Gwei), Nonstable	2,643,290	156.859	334.343	58.000	122.000	249.000
Absolute Value of Log Spot Rate, All	2,859,992	5.264	2.530	0.395	6.552	7.432
Absolute Value of Log Spot Rate, Stable	216,702	0.004	0.006	0.001	0.003	0.008
Absolute Value of Log Spot Rate, Nonstable	2,643,290	5.696	2.114	2.336	6.789	7.442

Variables

- **Token Exchange Rate Volatility**: standard deviation of the log spot rate between two tokens deposited in the AMM, in each week.
- **Deposit Flow Rate**:

$$Depositflow_{jt} = sgn(DepositA_{jt}) \times \left(\frac{DepositA_{jt}}{TokenA_{jt}} \times \frac{DepositB_{jt}}{TokenB_{jt}} \right)^{1/2}$$

- $DepositA_{jt}$, $DepositB_{jt}$ are the total tokens A and token B by liquidity providers of AMM j during week t
- $TokenA_{jt}$, $TokenB_{jt}$ are the initial deposits in AMM j during week t.

Variables

- **Trading Volume (of Investors)**:

$$Volume_{jt} = \left(\frac{TradeA_{jt}}{TokenA_{jt}} \times \frac{TradeB_{jt}}{TokenB_{jt}} \right)^{1/2},$$

  - $TradeA_{jt}$, $tradeB_{jt}$ are the total token A and token B traded by the investors with swap orders at AMM j in week t.

- **Gas Price Volatility:** the standard deviation of the gas price attached to all transactions executed on AMM j in week t.

Results

	Dependent variable: Deposit Inflow Rate		
	(a)	(b)	(c)
Intercept	0.067	-0.090^{**}	-0.016
	(0.066)	(0.042)	(0.042)
Exchange Rate Volatility	-0.410^{**}		-1.636^{***}
	(0.172)		(0.376)
Trading Volume		0.041^{***}	0.061^{***}
		(0.011)	(0.014)
Week fixed effects?	yes	yes	yes
AMM fixed effects?	yes	yes	yes
Observations	588	588	588
R^2	0.11	0.17	0.23
Note:			$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Result

- Negative and statistically significant relationship between volatility of token exchange rate and deposit flow rate
- Positive and statistically significant relationship between trading volume and deposit flow rate
- Consistent with our theoretical prediction that $\rho_1, \rho_2 < 0$, $\delta_1, \delta_2 > 0$.
- A one-standard-deviation increase in weekly spot rate volatilitydecreases the deposit flow rate by 10% standard deviations.
- A one-standard-deviation increase in trading volume increases deposit flow rate by 35% standard deviation.

Result

	Dependent variables:	
	Gas Price Volatility	Gas Price
	(a)	(b)
Intercept	218.994***	126.324***
	(29.344)	(2.975)
Stable	-81.933***	-13.4223***
	(17.040)	(3.022)
Week fixed effects?	yes	no
Day fixed effects?	no	yes
Protocol fixed effects?	yes	yes
Observations	588	2,860,041
R^2	0.13	0.04
Note:	*p<0.1; **p<0.05; ***p<0.01	

Result

- The gas price volatility for “stable pairs” is significantly lower than for “non-stable pairs”: around 40% lower;
- The gas price for “stable pairs” is significantly lower than for “non-stable pairs”: around 10% lower
- Consistent with our theoretical prediction that $\kappa_A < 0, \kappa_B < 0$.

Abadi, J. and Brunnermeier, M. (2018), Blockchain Economics, NBER Working Papers 25407, National Bureau of Economic Research, Inc.

Biais, B., Bisière, C., Bouvard, M. and Casamatta, C. (2019), 'The Blockchain Folk Theorem', *The Review of Financial Studies* **32**(5), 1662–1715.

Campbell, R. H. (2016), Cryptofinance, Working paper.

Capponi, A., Olafasson, S. and Alsabah, H. (2021), 'Proof-of-Work Cryptocurrencies: Does Mining Technology Undermine Decentralization?', *Major Revision at Management Science* .

Cong, L. W., Li, Y. and Wang, N. (2020), Token-Based Platform Finance, NBER Working Papers 27810, National Bureau of Economic Research, Inc.

Dai, M., Jiang, w., Kou, S. and Qin, C. (2021), 'Bitcoin Mining and Electricity Consumption', *Working Paper* .

Easley, D., Maureen, O. and Basu, S. (2019), 'From mining to markets: The evolution of bitcoin transaction fees', *Journal of Financial Economics* **134**(1), 91–109.

Glosten, L. R. (1994), 'Is the electronic open limit order book inevitable?', *The Journal of Finance* **49**(4), 1127–1161.

Glosten, L. R. and Milgrom, P. R. (1985), 'Bid, ask and transaction prices in a specialist market with heterogeneously informed traders', *Journal of Financial Economics* **14**(1), 71–100.

Lehar, A. and Parlour, C. (2021), Decentralized exchanges, Working paper.

Park, A. (2021), The Conceptual Flaws of Constant Product Automated Market Making, Working paper.

Prat, J. and Walter, B. (2018), An Equilibrium Model of the Market for Bitcoin Mining, CESifo Working Paper Series 6865, CESifo.
URL: *https://ideas.repec.org/p/ces/ceswps/$_6$865.html*

Reppen, M., Sircar, R. and Zongxi, L. (2019), 'A Mean Field Games Model for Cryptocurrency Mining'.

Sockin, M. and Xiong, W. (2020), A model of cryptocurrencies, Working Paper 26816, National Bureau of Economic Research.

Xu, J., Vavryk, N., Paruch, K. and Cousaert, S. (2021), 'Sok: Decentralized exchanges (dex) with automated market maker (AMM) protocols'.

Yermack, D. (2017), 'Corporate Governance and Blockchains', *Review of Finance* **21**(1), 7–31.