

Quantifying the Impact of Impact Investing

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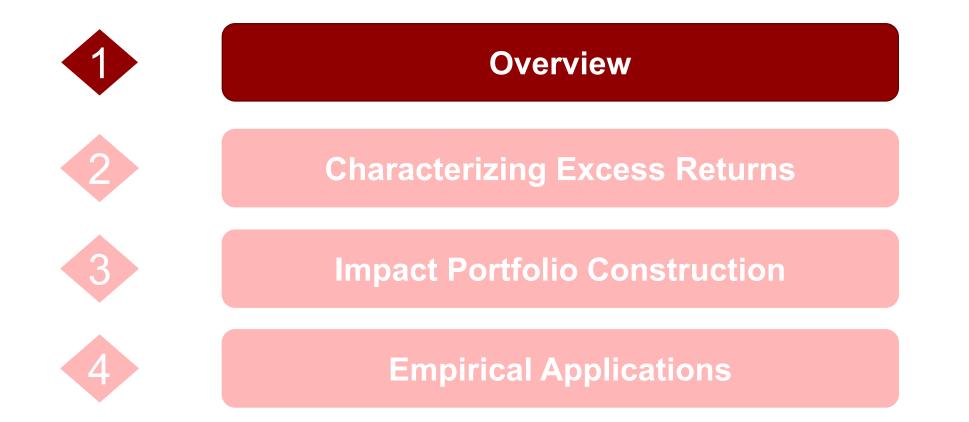
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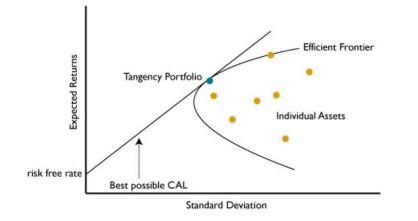


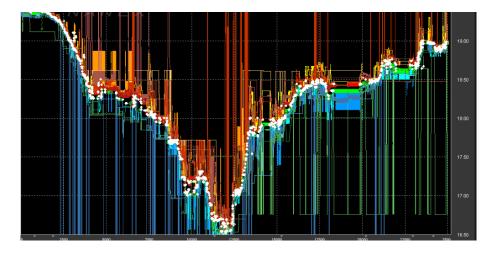
The Evolving Investment Landscape

□ From Warrant Buffet to Jim Simons

- Modern Portfolio Theory (Markowitz)
- Alpha Beta and Factor Models (Sharpe, Fama)
- Derivatives (Black-Scholes-Merton)
- Automation (Quant trading, HFT)

□ All of them focus on the **Monetary Outcome** of an investment







Impact Investing: investing for purposes other than solely to maximize riskadjusted returns

Maximize risk-adjusted returns, subject to constraints that supports certain social priorities and agendas

- Socially Responsible Investing (SRI)
- Environmental, Social, and Governance (ESG)
- Fossil fuels, tobacco, "sin stocks"
- Biotech and rare disease drugs





IN PARTNERSHIP WITH ITALY

PARIS2015 UN CLIMATE CHANGE CONFERENCE COP21·CMP11





The Royal Swedish Academy of Sciences has decided to award the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2018 to

"for integrating climate change into long-run macroeconomic analysis"



What About Fiduciary Duty?



WHAT IS THE SIGNIFICANCE OF BEING A FIDUCIARY?

Fiduciaries have important responsibilities and are subject to standards of conduct because they act on behalf of participants in a retirement plan and their beneficiaries. These responsibilities include:

- Acting solely in the interest of plan participants and their beneficiaries and with the exclusive purpose of providing benefits to them;
- Carrying out their duties prudently;
- Following the plan documents (unless inconsistent with ERISA);
- Diversifying plan investments; and
- Paying only reasonable plan expenses.

The duty to act prudently is one of a fiduciary's central responsibilities under ERISA. It requires expertise in a variety of areas, such as investments. Lacking that expertise, a fiduciary will want to hire someone with that professional knowledge to carry out the investment and other functions. Prudence focuses on the *process* for making fiduciary decisions. Therefore, it is wise to document decisions and the basis for those decisions. For instance, in hiring any plan service provider, a fiduciary may want to survey a number of potential providers, asking for the same information and providing the same requirements. By doing so, a fiduciary can document the process and make a meaningful comparison and selection.

Does impact investing meet this test?
 Does impact investing necessarily mean lower risk-adjusted returns?
 Framework?

LFE How To Evaluate Impact Investing?

□ Given a portfolio $W \equiv \sum_{i=1}^{N} \omega_i R_i$, traditional mean-variance optimization: $\max_{\{\omega_1,...,\omega_N \in \Omega\}} E[U(W)]$

□Consider any arbitrary subset $S \subseteq \{1,...,N\}$, and portfolios formed on the subset: $W^c \equiv \sum_{i \in S} \omega_i^c R_i$.

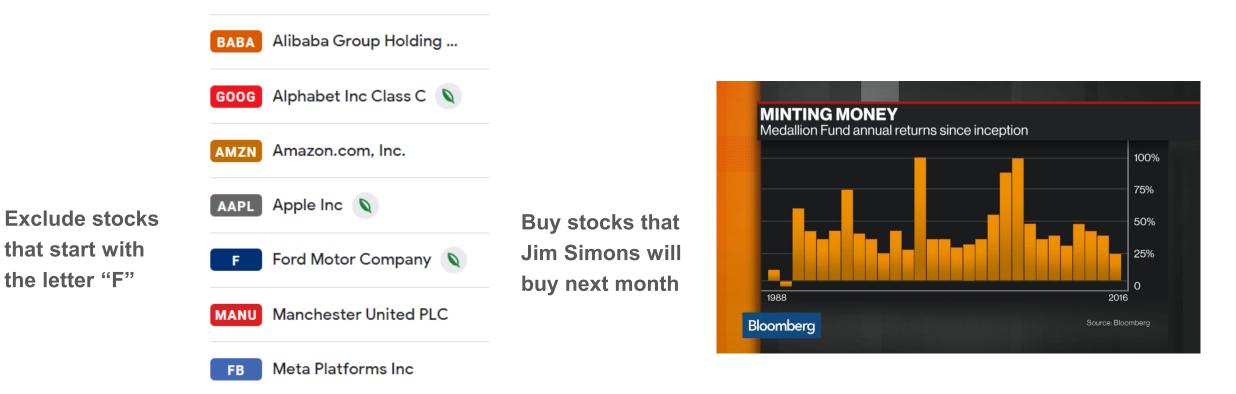
Then:

$$\max_{\{\omega_1,\ldots,\omega_N\in\Omega\}} E[U(W)] \ge \max_{\{\omega_i^c\in\Omega^c:\,i\in S\}} E[U(W^c)]$$

- Constrained optimization
- Worse risk-adjusted returns?



Consider two examples



Key: whether the subset S contains any information?
 i.e. Are constraints independent of returns?



□ A systematic framework to quantify the (financial) impact of impact investing.

- Main statistical tool: Induced order statistics
- Also called concomitants of the order statistic
- First applied to financial data by Lo and MacKinlay (1990) in a different context

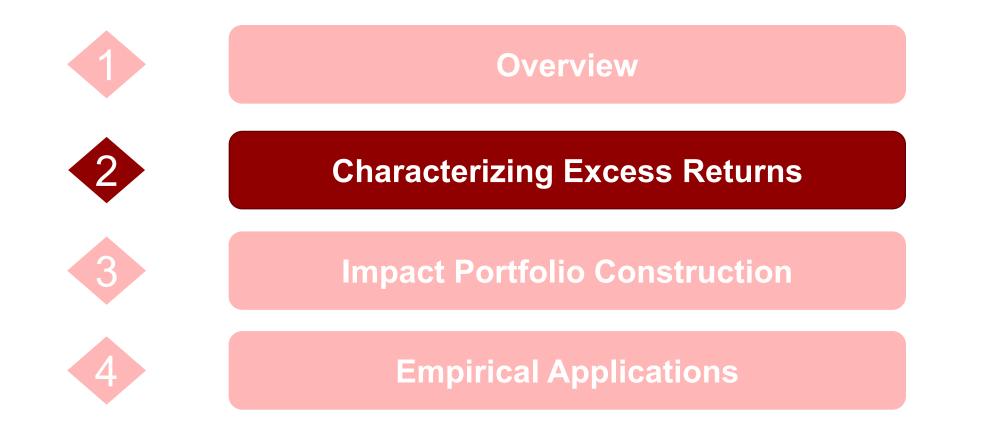
□ How to apply it optimally? Treynor-Black portfolios

□ Empirical demonstration in 4 examples

- Venture Philanthropy
- ESG
- Sin Stocks
- "Meme" stocks (e.g. Gamestop)

Related Literature







Basic linear multi-factor model: N securities with returns R_{it}

$$R_{it} - R_{ft} = \alpha_i + \beta_{i1} (\Lambda_{1t} - R_{ft}) + \dots + \beta_{iK} (\Lambda_{Kt} - R_{ft}) + \epsilon_{it}$$

such that $E[\epsilon_{it}|\Lambda_{kt}] = 0$, for k = 1, ..., K

 $\Box R_{ft}$: risk free rate $\Box \Lambda_{kt}$: *k*-th factor return $\Box \alpha_{i}$: excess return $\Box \beta_{ik}$: factor betas $\Box \epsilon_{it}$: idiosyncratic risk

Consistent with CAPM, ICAPM, APT, FF iff α_i = 0.
 Prop 1: If α_i = 0, impact investing can only make investors worse off financially.

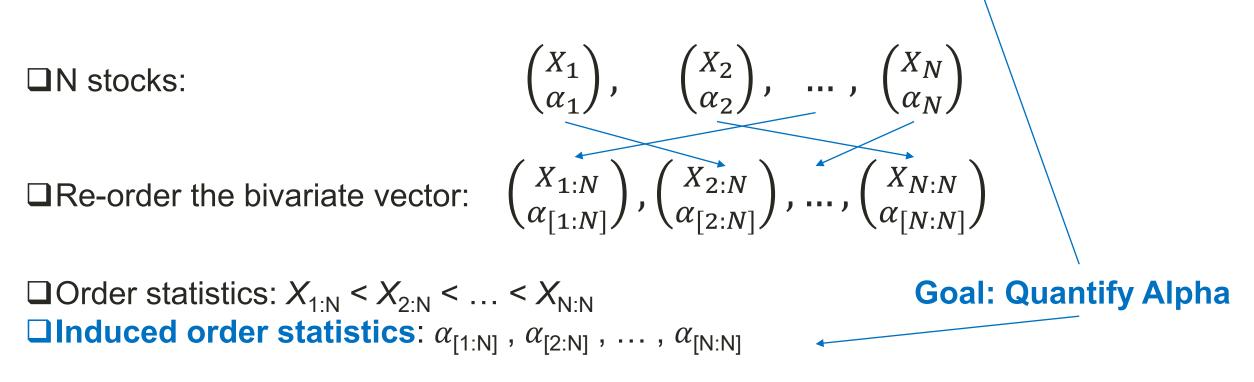
Q Suppose for simplicity that $\alpha_i \sim N(0, \sigma_{\alpha}^2)$



 \Box Impact factor used to construct portfolios: **X** = [$X_1, X_2, ..., X_N$]^T

□Impact Portfolio: any portfolio *S*(**X**) formed as a function of the impact factor, **X**.

- Selecting top n securities based on X
- Overweight top securities based on X

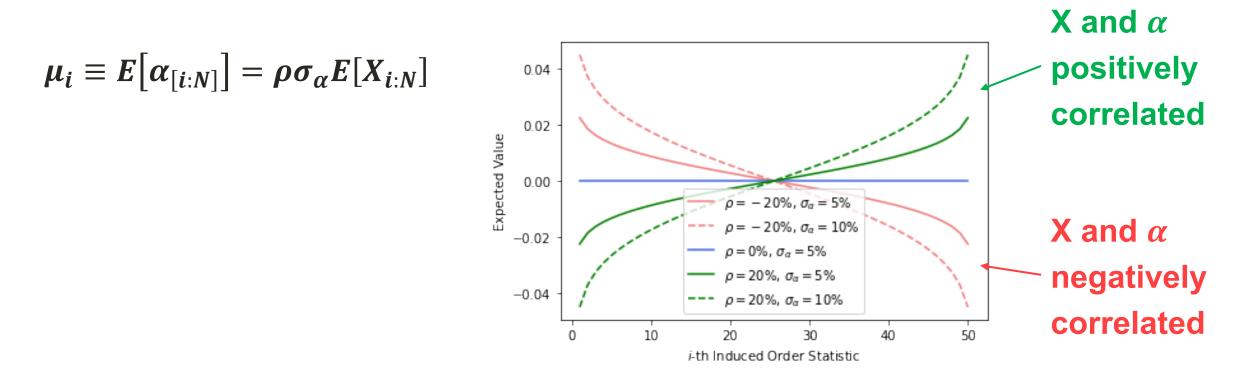




Expected Value of Induced Order Statistics (Prop 2)

□Suppose we have:
$$(X_1, \alpha_1), ..., (X_N, \alpha_N) \stackrel{\text{IID}}{\sim} N\left(\begin{bmatrix} \mu_x \\ \mu_\alpha \end{bmatrix}, \begin{bmatrix} \sigma_x^2 & \rho\sigma_x\sigma_\alpha \\ \rho\sigma_x\sigma_\alpha & \sigma_\alpha^2 \end{bmatrix}\right)$$

□Suppose W.L.O.G. that $\mu_x = \mu_\alpha = 0$ and $\sigma_x = 1$, then:





Sketch of Proof

Because $(X_i, \alpha_i)^T$, i = 1, 2, ..., N, are IID jointly normal, we can write: $\alpha_i = \mu_{\alpha} + \rho \frac{\sigma_{\alpha}}{\sigma_{\chi}} (X_i - \mu_X) + \epsilon_i$

where X_i and ϵ_i are mutually independent. □Ordering on X_i we have

$$\alpha_{[i:N]} = \mu_{\alpha} + \rho \frac{\sigma_{\alpha}}{\sigma_{x}} (X_{i:N} - \mu_{x}) + \epsilon_{[i]}$$

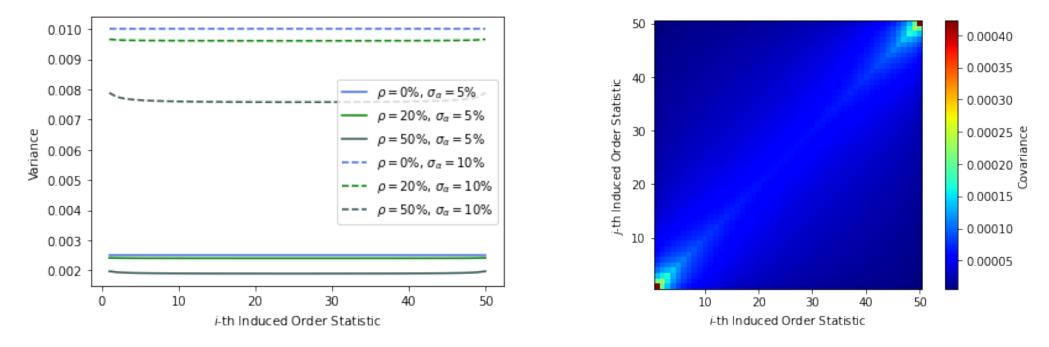
□ In view of the independence of X_i and ϵ_i , $X_{[i:N]}$ and $\epsilon_{[i]}$ are also independent. In particular, for expected value we have:

$$E[\alpha_{[i:N]}] = \mu_{\alpha} + \rho \sigma_{\alpha} E\left[\frac{X_{i:N} - \mu_{\chi}}{\sigma_{\chi}}\right] = \rho \sigma_{\alpha} E[Y_{i:N}]$$



Variance and Covariances of Induced Order Statistics

$$\begin{split} \sigma_i^2 &\equiv \operatorname{Var}\left(\alpha_{[i:N]}\right) = \sigma_\alpha^2 \left(1 - \rho^2 + \rho^2 \operatorname{Var}\left(\frac{X_{i:N} - \mu_x}{\sigma_x}\right)\right) = \sigma_\alpha^2 \left(1 - \rho^2 + \rho^2 \operatorname{Var}\left(Y_{i:N}\right)\right).\\ \sigma_{ij} &\equiv \operatorname{Cov}\left(\alpha_{[i:N]}, \alpha_{[j:N]}\right) = \sigma_\alpha^2 \rho^2 \operatorname{Cov}\left(\frac{X_{i:N} - \mu_x}{\sigma_x}, \frac{X_{j:N} - \mu_x}{\sigma_x}\right) = \sigma_\alpha^2 \rho^2 \operatorname{Cov}\left(Y_{i:N}, Y_{j:N}\right). \end{split}$$



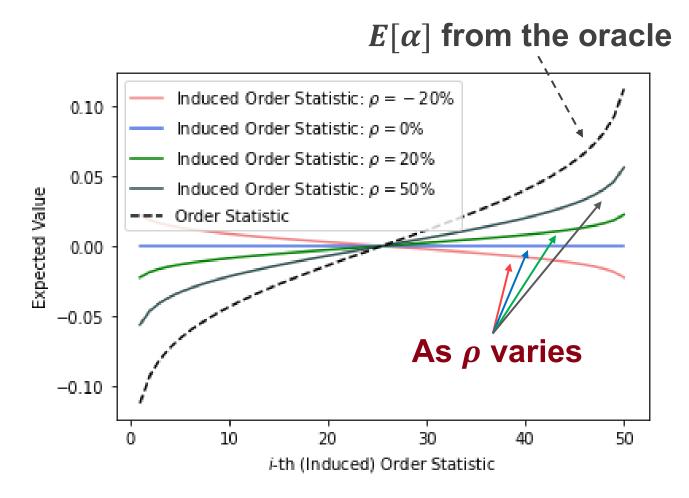
LFE Comparison with Conventional Order Statistics

Imagine we have an **omniscient oracle** that ranks securities by **unobserved alpha**

 $\mu_{i} \equiv E[\alpha_{[i:N]}] = \rho E[\alpha_{i:N}]$ $Var(\alpha_{[i:N]}) - \sigma_{\alpha}^{2} = \rho^{2}[Var(\alpha_{i:N}) - \sigma_{\alpha}^{2}]$

$$\operatorname{Cov}(\alpha_{[i:N]}, \alpha_{[j:N]}) = \rho^2 \operatorname{Cov}(\alpha_{i:N}, \alpha_{j:N})$$

(Prop 4) Induced order statistics is just a discounted version of the corresponding alpha from the omniscient oracle, where the discount is just p.



LFE Characterizing Alpha: Asympotics

Asymptotic distribution when $N \to \infty$ (Prop 5-6) \Box Assuming $(X_i, \alpha_i)^T$, i = 1, 2, ..., N, are IID, for any sequence $1 < i_1 < \cdots < i_n < N$ such that, as $N \to \infty$, $i_k/N \to \xi_k \in (0,1)$ for k = 1, ..., n, we have: $\lim_{N \to \infty} P(\alpha_{[i_1:N]} < a_1, \cdots, \alpha_{[i_n:N]} < a_n) = \prod_{k=1}^{n} P(\alpha_k < a_k | F_x(X_k) = \xi_k)$

k=1

where $F_{\chi}(\cdot)$ is the marginal CDF of X_i .

□ Moreover, when $(X_i, \alpha_i)^T$ are jointly normal, $\mu(\xi_k) \equiv \rho(\sigma_\alpha/\sigma_x)[F_x^{-1}(\xi_k) - \mu_x] = \rho\sigma_\alpha \Phi^{-1}(\xi_k)$ $\sigma^2(\xi_k) \equiv \sigma_\alpha^2(1 - \rho^2)$

□ Expected value depends on ρ , σ_{α}^2 , and the inverse CDF of "order" □ Variance is a constant across all orders □ Mutually independent

Interpreting Alpha as Omitted Factors

 $\alpha_i \neq 0$ can be interpreted as omitted factors (Prop 7):

$$R_{it} - R_{ft} = \chi_{i} + \beta_{i1} (\Lambda_{1t} - R_{ft}) + \dots + \beta_{iK} (\Lambda_{Kt} - R_{ft}) + \epsilon_{it}$$

Observed Omitted Factors $\longleftarrow \lambda_{i}$

□ No arbitrage or inefficiencies assumed

□ Impact PMs provide investors with exposure to novel factor risk premia

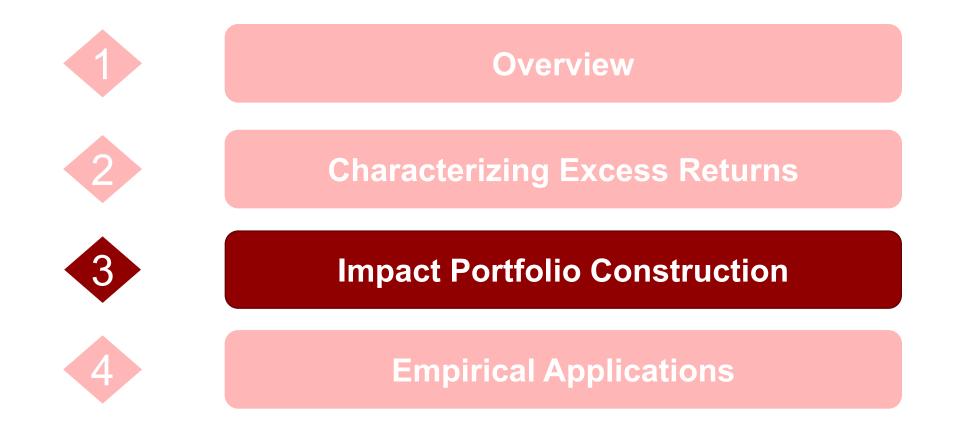
 \Box We generalize our results by introducing dependence *across* (X_i , λ_i)

$$E[\lambda_{[i:N]}] = \rho_{adj}\sigma_{\lambda}E[X_{i:N}]$$

$$\Box \rho_{adj} \equiv (\rho_{x,\lambda} - \tilde{\rho}_{x,\lambda})/(1 - \rho_x)$$

$$\Box \rho_{x,\lambda} \equiv Corr(X_i, \lambda_i), \quad \tilde{\rho}_{x,\lambda} \equiv Corr(X_i, \lambda_j), \quad \rho_x \equiv Corr(X_i, X_j)$$







With these results, we can quantify any impact portfolio (Prop 8)

$$\mathbf{E}[\alpha_p] = \sum_{i=1}^n \omega_i \mathbf{E}[\alpha_{[i:N]}] \quad , \quad \mathbf{Var}[\alpha_p] = \sum_{i=1}^n \sum_{j=1}^n \omega_i \omega_j \mathbf{Cov}\left[\alpha_{[i:N]}, \alpha_{[j:N]}\right]$$

The standard tools apply

- Sharpe, Sortino, information ratios
- Performance attribution
- Risk management, etc.

□Equal weighted Portfolios provide a way to estimate ρ and σ_{α}^2 .

$$E[\tilde{\alpha}] = \frac{1}{n_0} \sum_{i \in S} \mu_i = \frac{\rho \sigma_{\alpha}}{n_0} \sum_{i \in S} E[Y_{i:N}]$$



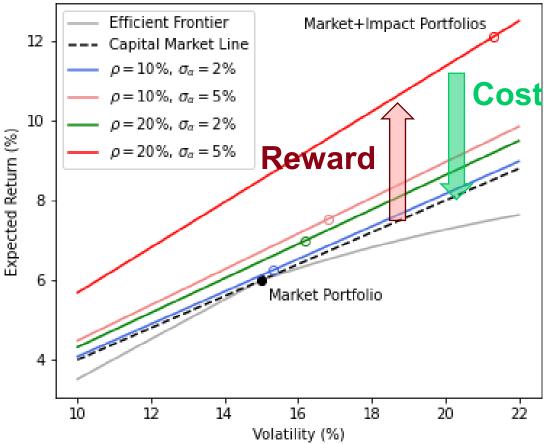
Treynor-Black show how to construct superefficient portfolios (Prop 9):

$$\omega_i^* \propto \frac{\mathrm{E}[\alpha_{[i:N]}]}{\sigma_i^2 + \sigma(\epsilon_i)^2}$$

Optimal asset allocation with a passive index portfolio (Prop 10)

A natural definition of the financial reward / cost of impact investing

❑As we vary the desired level of impact, this corresponds to the "ESG-efficient frontier" of Pedersen et al (2021) in an explicit way.





Impact Portfolio Alpha

Combining Impact portfolio with Passive portfolios

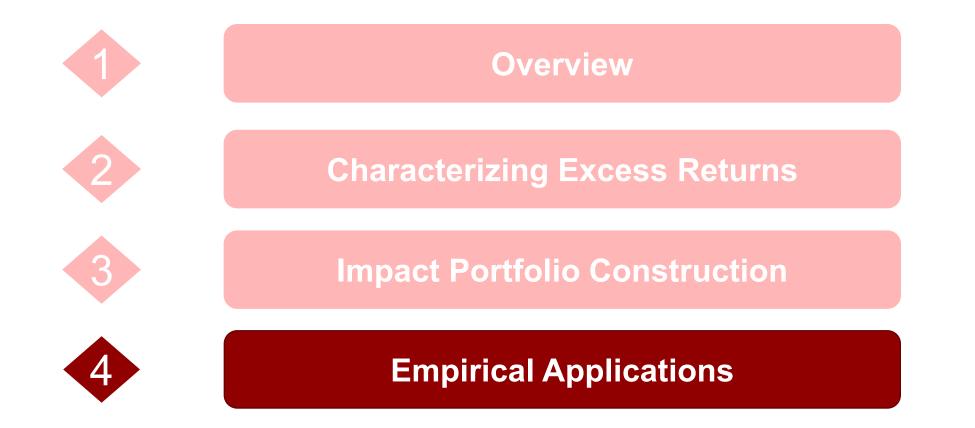
Completion ω_A				Expected Excess Return α_A				
Correlation ρ	Bottom	2^{nd}	9^{th}	Top	Bottom	2^{nd}	9^{th}	Top
			$\sigma_{\alpha} =$	1%				
$30\%~(R^2=9\%)$	-0.66	-0.39	0.39	0.66	-0.6%	-0.3%	0.3%	0.6%
$10\%~(R^2 = 1\%)$	-0.22	-0.13	0.13	0.22	-0.2%	-0.1%	0.1%	0.2%
$-10\%~(R^2 = 1\%)$	0.22	0.13	-0.13	-0.22	0.2%	0.1%	-0.1%	-0.2%
$-30\%~(R^2 = 9\%)$	0.66	0.39	-0.39	-0.66	0.6%	0.3%	-0.3%	-0.6%
			$\sigma_{\alpha} =$	2%				
$30\%~(R^2=9\%)$	-1.31	-0.78	0.78	1.31	-1.1%	-0.6%	0.6%	1.1%
$10\%~(R^2=1\%)$	-0.44	-0.26	0.26	0.44	-0.4%	-0.2%	0.2%	0.4%
$-10\% \ (R^2 = 1\%)$	0.44	0.26	-0.26	-0.44	0.4%	0.2%	-0.2%	-0.4%
-30% $(R^2 = 9\%)$	1.31	0.78	-0.78	-1.31	1.1%	0.6%	-0.6%	-1.1%
			$\sigma_{lpha} =$	5%				
$30\%~(R^2=9\%)$	-3.23	-1.93	1.93	3.23	-2.8%	-1.6%	1.6%	2.8%
$10\% \ (R^2 = 1\%)$	-1.08	-0.64	0.64	1.08	-0.9%	-0.5%	0.5%	0.9%
$-10\% \ (R^2 = 1\%)$	1.08	0.64	-0.64	-1.08	0.9%	0.5%	-0.5%	-0.9%
$-30\% \ (R^2 = 9\%)$	3.23	1.93	-1.93	-3.23	2.8%	1.6%	-1.6%	-2.8%

 α_A $\omega_A =$

$E[R_m]$	$-R_{f} =$	6%
$\sigma_m = 15$	5%	

Even small correlations lead to meaningful alpha







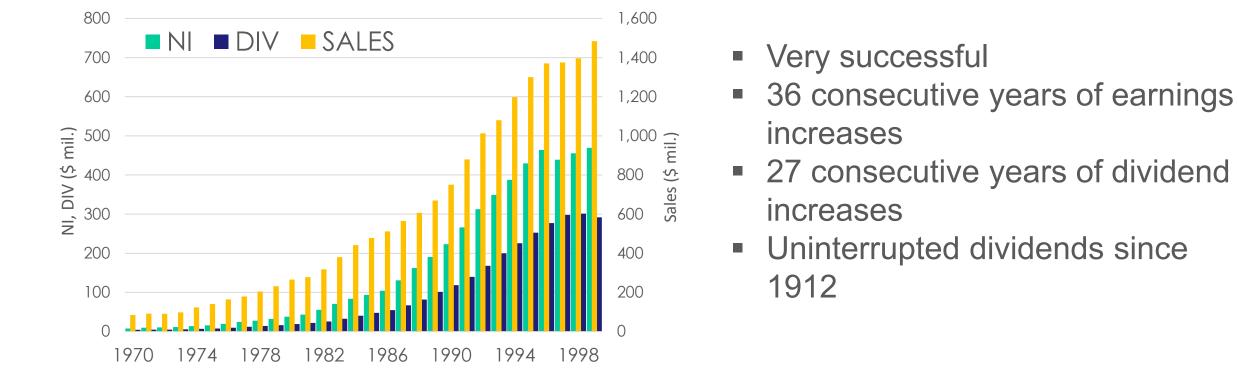
Sin stocks
ESG
Venture philanthropy
"Meme" stocks (e.g. Gamestop)



□Sin stocks: stocks from companies involved in unethical or immoral activities.

Examples: alcohol, tobacco, gambling, sex-related industries, and firearms.

□U.S. Tobacco as of Dec 1998 (HBS Case study 9-200-069)





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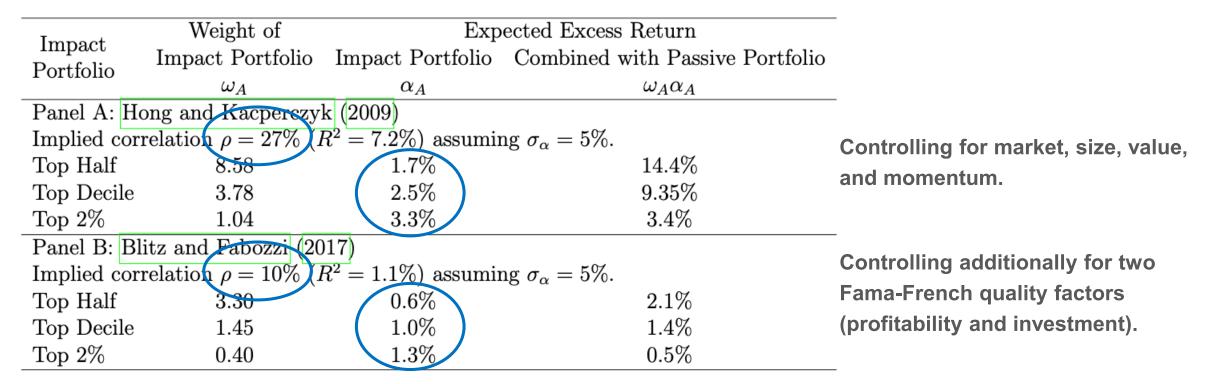
							1
UST	Phillip Morris	N orth Atlantic	RJR Nabisco	Standard Commercial	DiMon	Universal	Average (other)
80.1%	41.7%	65.4%	46.2%	12.3%	9.7%	14.3%	31.6%
32.9%	10.3%	1.1%	3.5%	2.4%	1.8%	3.0%	3.7%
103.4%	49.3%	N/A	8.4%	12.5%	22.5%	25.6%	23.7%
53.8%	13.2%	40.0%	2.4%	2.7%	3.4%	6.5%	11.4%
17.6%	47.5%	90.0%	54.4%	71.9%	72.3%	59.4%	65.9%
1.5%	10.1%	N/A	49.8%	69.3%	67.4%	38.7%	47.1%
	11.2	1.3	2.5	3.3	2.6	3.5	4.1
	A	B+	BBB-	BB-	BB+	A-	
	80.1% 32.9% 103.4% 53.8% 17.6%	UST M orris 80.1% 41.7% 32.9% 10.3% 103.4% 49.3% 53.8% 13.2% 17.6% 47.5% 1.5% 10.1%	USTMorrisAtlantic80.1%41.7%65.4%32.9%10.3%1.1%103.4%49.3%N/A53.8%13.2%40.0%17.6%47.5%90.0%1.5%10.1%N/A11.21.3	USTM orrisAtlanticN abisco80.1%41.7%65.4%46.2%32.9%10.3%1.1%3.5%103.4%49.3%N/A8.4%53.8%13.2%40.0%2.4%17.6%47.5%90.0%54.4%1.5%10.1%N/A49.8%1.5%11.21.32.5	USTMorrisAtlanticNabiscoCommercial80.1%41.7%65.4%46.2%12.3%32.9%10.3%1.1%3.5%2.4%103.4%49.3%N/A8.4%12.5%53.8%13.2%40.0%2.4%2.7%17.6%47.5%90.0%54.4%71.9%1.5%10.1%N/A49.8%69.3%11.21.32.53.3	USTMorrisAtlanticNabiscoCommercialDiMon80.1%41.7%65.4%46.2%12.3%9.7%32.9%10.3%1.1%3.5%2.4%1.8%103.4%49.3%N/A8.4%12.5%22.5%53.8%13.2%40.0%2.4%2.7%3.4%17.6%47.5%90.0%54.4%71.9%72.3%1.5%10.1%N/A49.8%69.3%67.4%11.21.32.53.32.6	USTMorrisAtlanticNabiscoCommercialDiMonUniversal80.1%41.7%65.4%46.2%12.3%9.7%14.3%32.9%10.3%1.1%3.5%2.4%1.8%3.0%103.4%49.3%N/A8.4%12.5%22.5%25.6%53.8%13.2%40.0%2.4%2.7%3.4%6.5%17.6%47.5%90.0%54.4%71.9%72.3%59.4%1.5%10.1%N/A49.8%69.3%67.4%38.7%1.121.32.53.32.63.5



□ Sin stocks: stocks from companies involved in unethical or immoral activities.

Examples: alcohol, tobacco, gambling, sex-related industries, and firearms.

□Correlation(Sin Stocks, Alpha): 10-27%. □Financial Cost of Divesting Sin Stocks: 1-3% per annum.



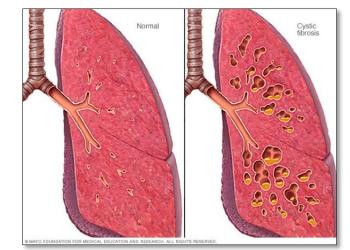


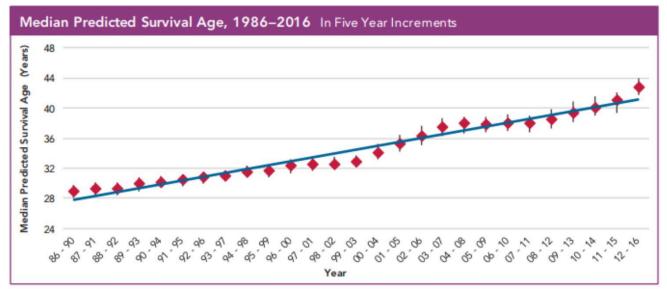
- Correlation ranges from -2% to 22% in these examples, implying financial reward or cost of ESG investing, in various forms.
- Depends on specific metric, asset class, region, time period, etc.
- Disclose the correlations, especially when they are negative.

Impost	Weight of	Exp	pected Excess Return
Impact Portfolio	Impact Portfolio	Impact Portfolio	Combined with Passive Portfolio
FOLIOIO	ω_A	α_A	$\omega_A \alpha_A$
Panel A: M			
Implied cor	relation $\rho = 1.6\%$ (1		g $\sigma_{\alpha} = 5\%$.
Top Half	0.11	0.10%	0.01%
Top Decile	0.05	0.15%	0.00%
Top 2%	0.01	0.20%	0.00%
	aker et al (2021)		
Implied cor	relation $\rho = -2.0\%$	$(R^2 = 0.04\%)$ assum	ning $\sigma_{\alpha} = 1\%$.
Top Half	-1.86	-0.02%	-0.03%
Top Decile	-0.47	-0.04%	-0.02%
Top 2%	-0.13	-0.05%	-0.00%
Panel C: Ba	ansal, Wa, and Yar		
Implied cor	relation $\rho = 22\%$ (R	$R^2 = 4.7\%$) assuming	$\sigma_{\alpha} = 5\%.$
Top Half	2.69	1.35%	3.64%
Top Decile	1.19	1.99%	2.37%
Top 2%	0.33	2.65%	0.88%
Panel D: Ba	ansal, Wu, and Yax	on (2021) ("bad tir	mes")
Implied cor	relation $\rho = -0.2\%$	$R^2 = 0.0\%$) assumi	ing $\sigma_{\alpha} = 5\%$.
Top Half	-0.02	-0.01%	-0.00%
Top Decile	-0.01	-0.02%	-0.00%
Top 2%	-0.00	-0.02%	-0.00%



- Venture Philanthropy: non-profit organizations combined with venture capital practices
- A case study of the Cystic Fibrosis (CF) Foundation (Kim and Lo 2019)
- □ Rare genetic disease
- Progressive, multi-system disease; respiratory failure causes death
- □ Life expectancy of 28 years in 1986

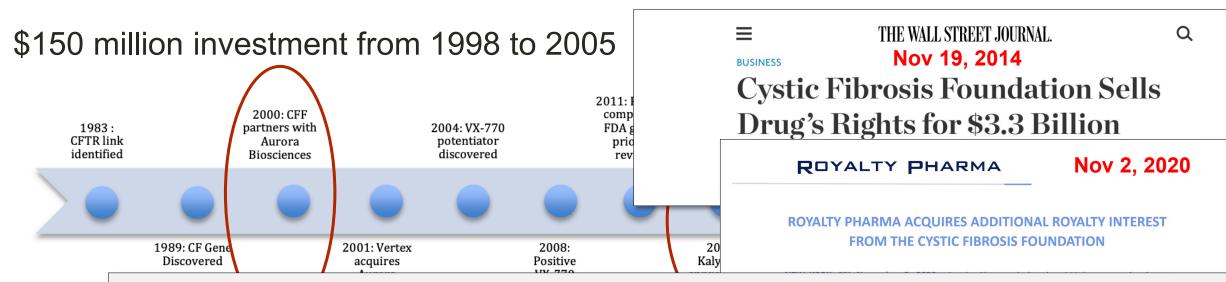




*Using the currently recommended method for calculating median predicted survival.

Figure 1. Median Predicted Survival Age of CF Patients over Time. Source: Cystic Fibrosis Foundation's 2016 Patient Registry Annual Data Report





"The mission of the Cystic Fibrosis Foundation is to cure cystic fibrosis and to provide all people with the disease the opportunity to lead full, productive lives by funding research and drug development, promoting individualized treatment and ensuring access to high-quality, specialized care."

Then... Why Don't People Invest?



"Not all impact investments are created equal": A new framework to quantify and manage the financial impact of impact investing

- \Box Know your ρ , and disclose it to investors
- How to optimally exploit impact alpha
- □More broadly applicable to any characteristics correlated with returns
- **Comments welcome:** https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3944367

Doing Well by Doing GoodFinance does not have to be a zero-sum game!



Thank you!