Implementing Momentum Strategy with Options: Dynamic Scaling and Optimization

Abstract: Momentum strategy and its option implementation are studied in this paper. Four basic strategies are constructed with dynamically adjusted scaling factors to attain a comparable risk level with respect to index. Three improvements are suggested, i.e., stop loss, index combined and long-short volatility. Further, dominant strategy switching between scenarios leads to the idea of performing mean-variance optimization on basic strategies. Compared with basic strategies, these enhancements yield better performance metrics.

Keywords: Momentum Strategy, Option, Scaling, Mean-Variance Optimization

CONTENTS

1. Introduction

Introduced by Jegadeesh and Titman in 1993, momentum strategy was defined as an investment strategy that aims to capitalize on the continuance of existing trends in the market. The existence of momentum profits in international markets of different securities was shown by Rouvenhorst(1998), Chan et al.(2000) and Assnes, Moskowitz and Pedersen(2013).

However, little evidence has shown that these momentum effects in security market can be realized by derivatives like options. Here, we will focus on the effectiveness of implementing momentum strategies with options.

Since options are leveraged, which means their cumulative returns will crash easily, we scale them by their lambda to achieve the same level of risk as index. Then, we compare performance of original index momentum strategy with that of scaled ones, including cumulative returns, VaR, CVaR, Sharpe Ratio, maximum drawdown and Greeks.

To improve portfolio performance, we introduce three modifications. One is to apply stop loss strategy by setting a specific maximum drawdown as threshold. The second one is to diversify original portfolios portfolio by allocating 1/3 of our asset into index, since they have negative beta with respect to the index. The last one is to utilize MA crossovers as signals to long/short volatility by buying/selling straddle, since there are higher market volatilities during downswings than upswings.

After carefully analyzing the return series of above mentioned strategies in different market scenarios, we discover that winner strategy is changing. Thus, we take our four constructed strategies as underlying assets and use the mean-variance optimization to get an optimal mixture with the highest Sharpe Ratio.

2. Strategy Construction

Before we start, we retrieve index price, risk-free rate (overnight index swap) and 3-month at-the-money implied volatility from 01/03/2007 to 12/29/2017 from Bloomberg Terminal. Throughout this paper, we utilized the Black Scholes Model to derive theoretical at-the-money option price each day, and assume there are 360 days in one year.

2.1 Strategy Description

Strategy 1 is an index momentum portfolio: long/short index when its 60-day average is higher/lower than its 120-day average with close price. The position is always exposed to the market, either long or short.

Strategy 2 is an option momentum portfolio: long 1 share of 90-day call/put option when 60-day average is higher/lower than its 120-day average on a daily basis, vice versa.

Strategy 3 is a straddle momentum portfolio: long 1 share of 90-day straddle, rebalancing on a daily basis.

Strategy 4 is an alternative straddle momentum portfolio: long 1 share of 90-day straddle at the end of day 1, sell it and buy 1 share of 89-day straddle at the end of day 2, and so forth until expiration, we rebalance the position.

2.2 Implementation Result

After implementing of four portfolios above, we have their cumulated returns as shown in Figure 1. Only Strategy 1 has similar trend to index, while others are highly volatile. Specifically, Strategy 2 and Strategy 4 crash at the first 2 years (2007-2009), while Strategy 3 soars at some specific periods but plunges later. The reason lays in the properties of options.

Constructed by straddle, Strategy 3 bets on volatile market, so it will perform well when the index either swings up quickly or crash, like 2007-2009. That's why its cumulated return reaches 700% in the first 2 years. For Strategy 2, Strategy 3 and Strategy 4, their returns are enlarged by the leverage effect options, causing drastic losses during plunge.

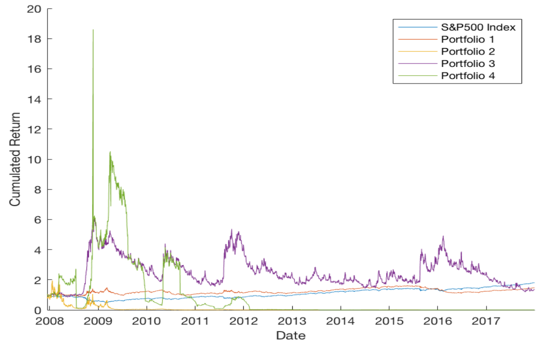

Figure 1 Implementation Result

Apparently, portfolios with high maximum drawdown are not what we want. We would like to scale our positions down to avoid bankruptcy in case of a bad run. In practice, traders prefer to use 5%-10% of assets as investment capital for any trade. In this paper, since the portfolio constructed by options provides more information such as lambda (Greeks), we could utilize the information to dynamically scale our portfolio.

2.3 Scaling Problems

In order to evaluate the effectiveness of the above portfolios, we first need to scale the risk down to the same level as trading the index.

That is to say, we need to determine what fraction of current wealth should be invested in the option portfolio such that after leverage, the return and risk would have the same order of magnitude as investing in index.

2.3.1 Scaling Parameter for Call & Put Option

This fraction is easy to solve for Strategy 2, which consists of either a call or a put, by using the option's lambda, λ. Lambda measures the percentage change of the option price with respect to the percent change in the underlying index, which is given by:

$$\lambda = \frac{\partial V}{\partial S} \times \frac{S}{V} = \Delta \times \frac{S}{V}$$

where S is the index level and V is the option price.

According to Black Scholes setting, lambda can be calculated as:

$$\lambda_{call} = \frac{N(d_+)\, S}{C}, \qquad \lambda_{put} = \frac{-N(-d_+)\, S}{P}$$

where C is call option price and P is put option price.

If we invest a fraction w of our money to an option, then the whole portfolio has a lambda:

$$w\lambda_{option}$$

In order for the portfolio to "mimic" the index, we want to find 2 scaling factors, w_{call} and w_{put} such that:

$$\begin{cases} w_{call}\lambda_{call} = \lambda_{index} = 1 \\ w_{put}\lambda_{put} = -\lambda_{index} = -1 \end{cases}$$

In other words, we want the option portfolio behaves like longing an index when we are holding calls and shorting an index when we are holding puts. Then we have:

$$\begin{cases} w_{call} = \dfrac{1}{\lambda_{call}} \\ w_{put} = -\dfrac{1}{\lambda_{put}} \end{cases}$$

2.3.2 Scaling Parameter for Straddle

For Strategy 3 and Strategy 4, the above formulas are not applicable since they use straddles instead of single calls or puts. Intuitively, we long a straddle to avoid guessing the direction of market move, just betting that the volatility will be high. If we consider each leg of the straddle, we can interpret this intuition as requiring the call leg to mimic a long position in index and the put leg to mimic a short position in index. By doing so, we are immune to the first order movement of market in both directions while earning money from Gamma and Vega. So, we want:

$$\begin{cases} w\, \dfrac{C}{C+P}\lambda_{call} = 1 \\ w\, \dfrac{P}{C+P}\lambda_{put} = -1 \end{cases}$$

where $\frac{C}{C+P}$ is the ratio of the call option in a straddle, $\frac{P}{C+P}$ is that of the put option.

However, these two equations cannot be satisfied simultaneously, so we should rather find a w that satisfies them as much as possible, i.e.

$$\min_{w} \left(w\, \frac{C}{C+P}\lambda_{call} - 1\right)^2 + \left(w\, \frac{P}{C+P}\lambda_{put} + 1\right)^2$$

By using the first order condition, we have:

$$2\left(\frac{wC}{C+P}\lambda_{call} - 1\right)\frac{C}{C+P}\lambda_{call} + 2\left(\frac{wP}{C+P}\lambda_{put} + 1\right)\frac{P}{C+P}\lambda_{put} = 0$$

$$w = \frac{(C\,\lambda_{call} - P\,\lambda_{put})(C+P)}{C^2\lambda_{call}^2 + P^2\lambda_{put}^2}$$

And the second order derivative:

$$\left(\frac{C}{C+P}\lambda_{call}\right)^2 + \left(\frac{P}{C+P}\lambda_{put}\right)^2 > 0$$

Shows that the solution is indeed a minimum.

2.4 Result

2.4.1 Backtesting Performance

We use these ratios to scale the return of portfolios involving options and keep Strategy 1 intact. (see Figure 2)

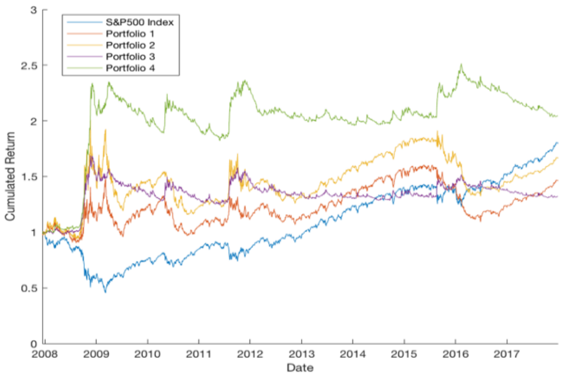

Figure 2 Cumulative Returns of Scaled Portfolios

Compared with the previous result, the scaled portfolios now have more stable cumulative returns. All portfolios now have a cumulative return of the same order of magnitude. As shown in Table 1, we further compare the Sharpe Ratios and maximum drawdowns (Max DD), in which information before scaling is shown in shadowed cells. It shows the efficiency of the scaling parameters, especially in terms of the maximum drawdown. The bankruptcy of Strategy 2, Strategy 3 and Strategy 4 we have seen before no longer exists.

Table 2 Strategy Return Statistics Comparison

	Sharp Ratio	Wealth	Max DD
Index	0.4432	1.7963	0.5483
Strat 1	0.3225	1.4606	0.3510
	0.3225	1.4606	0.3510
Strat 2	0.7709	0.0011	1.0000
	0.3763	1.6636	0.4253
Strat 3	0.3961	0.1049	0.9803
	0.3018	1.3197	0.2623
Strat 4	-0.4712	0.0000	1.0000
	0.7652	2.0423	0.2249

With respect to risk measures, (see Table 2), we find that among the four portfolios, Strategy 4 has the best performance, but each portfolio has their advantages – Strategy 2 has highest mean return while Strategy 4 has lower risks of rare events.

Table 2 Risk Measures

	Risk Measures	
	VaR	CVaR
Index	-7.0182	-11.5461
Strat 1	-6.7935	-11.1503
Strat 2	-6.8205	-12.6678
Strat 3	-3.1797	-4.9735
Strat 4	-2.3453	-3.7050

As is plotted in Figure 2, they all perform well during the sharp downturn in 2008-2010. However, they do not generate the same return streams. To be specific, Strategy 1 and Strategy 2 have basically similar trends, but Strategy 2 outperforms 1 throughout the period. Likewise, Strategy 4 outperforms Strategy 3 though sharing similar trends. Strategy 1, which simply uses moving averages of different terms as trading signals could avoid huge losses during the financial crisis, but the signal might be misleading during volatile market, like during 2016 and 2017. By contrast, Strategy 3 and Strategy 4 are better immunized from volatilities, as straddles are longing the volatility throughout the whole period. However, Strategy 3 turns out to be less profitable since over the last 10 years, when volatility was lower than usual. In Strategy 4, as we constantly rebalance by adjusting options' maturities, the return stream behaves much better than the others.

In the meantime, these portfolios exhibit negative correlation with respect to the index. This indicates that index could be a potential hedging asset for the four portfolios. Actually, it is an improved strategy we will talk about later.

2.4 Return Distributions

By plotting them in comparison with Strategy 1, we have a clearer look at the distributions of returns of the four portfolios. (see Figure 3)

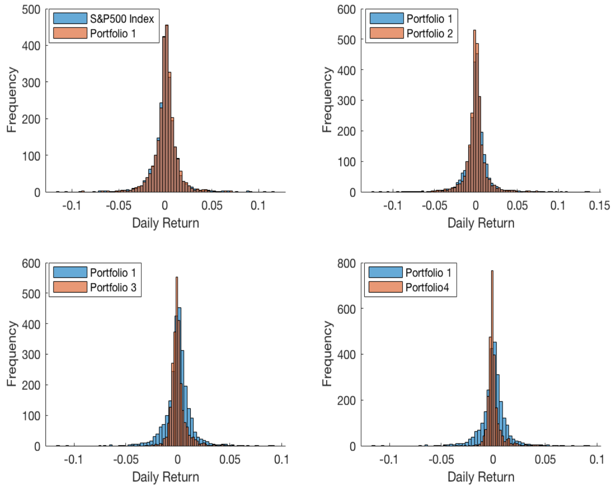

Figure 3 Return Distribution

In addition, we calculate the third moments for each portfolio in Table 3. Only Strategy 1 is negatively skewed, while the others are positively skewed. Namely, only Strategy 1 has long tails in the left, while others have long tails in the right.

Table 3 Return Distribution

Strategy	Distribution Condition		
	Mean	Standard Deviation	Skewness
Index	0.1087	0.2438	-0.0867
Strat 1	0.0786	0.2438	-0.1382
Strat 2	0.1087	0.2890	1.1499
Strat 3	0.0458	0.1518	1.9627
Strat 4	0.1050	0.1386	3.4940

3. Strategy Optimization

In order to realize a better performance, we introduce three improvements to ameliorate our portfolios.

3.1 Stop Loss Strategy

To improve our performance, we first provide a stop-loss strategy that limits the substantial downside risk of previous four strategies. The key idea is to close the current position in these strategies and invest in risk-free bond instead once a certain loss level is triggered, and to open the position when the market rebounds and reaches a certain high level.

We set the stop loss level at -10% of previous highest point and re-entry level at 10% of previous lowest point and find out our new strategies outperform the original ones.

In Figure 4 we can see that when the original strategy was about to plunge, the stop loss strategy is able to discern such tremendous loss signal and close the momentum position in time. (see also Table 4)

Figure 4 Performance Comparison for Strategy 1 with Strategy 1 (Stop Loss)

Table 4 Performance for Stop Loss Strategy

	Sharpe Ratio	Wealth	Max DD
Strat 1	0.3565	1.5010	0.2819
Strat 2	0.4541	1.9012	0.3184
Strat 3	0.3891	1.4434	0.2172
Strat 4	0.9493	2.3164	0.1924

3.2 Combined Strategy

The momentum strategy has attractive returns compared to a static buy-and-hold strategy, but it suffers from momentum crashes, especially during the market rallies from a sharp sell-off. Therefore, hedging momentum strategy with a static buy-and-hold strategy can achieve significant diversification of risk. It is justified by the negative correlation between the index and our four portfolios in Table 5.

Table 5 Correlation Matrix Between Index and Portfolio

	Index	Strat 1	Strat 2	Strat 3	Strat 4
Index	1.0000	-0.4065	-0.6147	-0.7377	-0.5085
Strat 1	-0.4065	1.0000	0.9507	0.1695	0.0559
Strat 2	-0.6147	0.9507	1.0000	0.4620	0.3072
Strat 3	-0.7377	0.1695	0.4620	1.0000	0.8501
Strat 4	-0.5085	0.0559	0.3072	0.8501	1.0000

Normally we should decide an optimal ratio to hedge our momentum strategy. In this case, in order to simplify our model and avoid overfitting, we decide to use a simple constant ratio, 1/3 instead of 1/2 to make momentum strategy dominant strategy and a positive return.

Take Strategy 1 as an example (Figure 5, Table 6), after using combined strategy, we can increase the performance. The same situation also applies to other portfolios. Therefore, combined momentum strategy is a good way to hedge our risk from the momentum crashes.

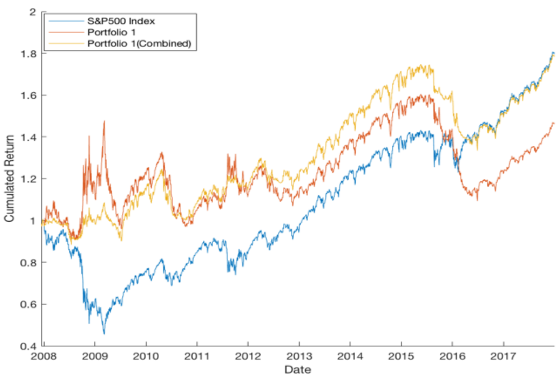

Figure 5 Performance Comparison for Strategy 1 with Strategy 1 (Combined)

Table 6 Performance for Combined Strategy

	Sharpe Ratio	Wealth	Max DD
Strat 1	0.5924	1.7834	0.2362
Strat 2	0.6936	2.0385	0.2188
Strat 3	0.9695	1.6272	0.1502
Strat 4	1.2317	2.1370	0.1331

3.3 Long-Short Volatility Strategy

In the previous part, we always long straddle as an instrument to implement momentum strategy. The underlying logic is that if the market increases much or decreases much, investors tend to believe that a trend exits, in which case longing straddle earn high return. From another perspective, however, straddle is actually a strategy that bets on volatility. Normally, volatility tends to rise when market crashes while remains relatively stable in bull market. Therefore, we could long straddle when 120-day moving average price is greater than 60-day's (bear market and high volatility) and vice versa (bull market and low volatility). This idea could be an adjustment to Strategy 3 and Strategy 4. As was expected, the modified Strategy 3 and 4 outperform the original ones (shown in Figure 6, Table 7).

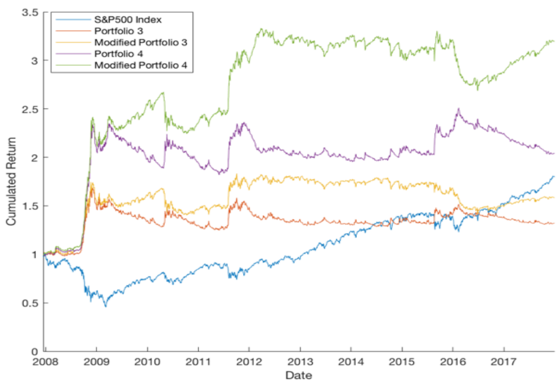

Figure 6 Performance Comparison for Modified Strategy 3&4 with Original Ones

Table 7 Return Comparison for Strategy 3&4

	Sharp Ratio	Wealth	Max DD
Index	0.4432	1.7963	0.5483
Strat 3	0.3018	1.3197	0.2623
Modified 3	0.4748	1.5865	0.2264
Strat 4	0.7652	2.0423	0.2249
Modified 4	1.2345	3.2062	0.1934

4. Dynamic Strategy

So far, we have improved the performance of given momentum strategy individually, we would like to construct a new portfolio comprised of them all.

4.1 Strategy Description

From the previous result, the return series of these strategies give us another view of market. Since each strategy has its own underlying logic, it would perform differently under different market scenarios (see Figure 7).

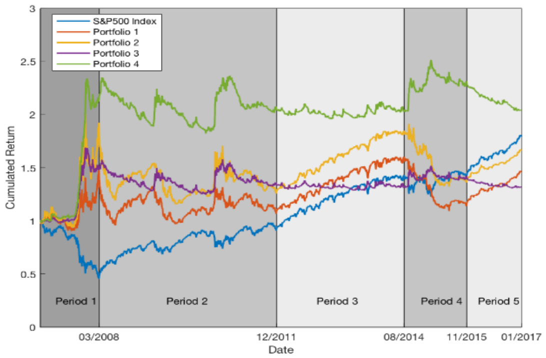

Figure 7 Scenario Partition

In order to verify our conjecture, we subsequently separate the 10-year period to 5 regions and calculate main indicators to show performance of each strategy (Table 8&9). For instance, we find out that the Strategy 2 outperforms others during period 1 and Strategy 3 outperforms others during period 2.

Table 8 Strategy Comparison (01/03/2007 - 03/26/2008)

	Sharpe Ratio	Wealth	Max DD
Strat 1	0.4934	1.0825	0.1270
Strat 2	1.2644	1.3303	0.1444
Strat 3	0.3591	1.0557	0.1447
Strat 4	1.7196	1.3331	0.1276

Table 9 Strategy Comparison (03/27/2009 - 12/05/2011)

	Sharpe Ratio	Wealth	Max DD
Strat 1	0.2140	1.0634	0.1973
Strat 2	0.0794	1.0052	0.1945
Strat 3	1.2375	1.2217	0.0475
Strat 4	1.2242	1.2647	0.0624

The fact that the dominated portfolio switches in different market scenario indicates that mean-variance framework could be applied to construct a ‘dynamic’ portfolio by regarding them as four tradable securities. By using Markowitz’s Modern Strategy Theory (MPT), we could have a close form formula for the optimal weight, i.e. the weight of “market portfolio”, which is given by:

$$w = \frac{\Sigma^{-1}(\mu - r_f I)}{B - A r_f}$$

$$A = I^T \Sigma^{-1} I,\ B = \mu^T \Sigma^{-1} I$$

where μ and Σ are respectively the expected return and covariance matrix of the properly scaled strategies, I is the unit column vector and r_f is the risk-free rate. However, the classical MPT solution has two main problems.

Firstly, since the classical MPT does not pose any constraint on each component of the weight vector, so if the covariance matrix is near singular, the weight of some strategy may have extremely big absolute value. In such case, if the next movement is opposite to our portfolio, we may experience unacceptable loss in one day. In backtesting, the dynamic portfolio will go bankrupt easily.

Secondly, under the extreme market condition that the minimum-risk portfolio on the efficient frontier has an expected return less than the risk-free rate, the closed form formula does not yield the optimal risky asset allocation. In such situation, it is optimal to short all risky securities and buy risk-free bonds. But the quantity of short selling cannot be determined without utility function or risk averse assumption.

To avoid these instabilities in our solution, we do the following maximization to find the optimal weight for each sub-strategy:

$$\max_{w} \frac{w^T \mu - r_f}{\sqrt{w^T \Sigma w}}$$

$$s.t. \quad w^T 1 \le 1,\ \ -1 < w_i < 1$$

Here we adjust the weight to maximize the expected Sharp Ratio, subject to the constraint of short selling of the four portfolios and restricted weights within a reasonable range in order to prevent bankruptcy. Also, we relax the constraint in the classical MPT that we must invest all money, and allow partial investment. This protect us from extreme conditions where all strategies perform poorly, in which case we put less money in stock market and invest the remainder in risk-free bonds.

In practice, we estimate these two statistics by calculating the sample mean and covariance matrix on the daily return series with a sliding window of 120 trading days. Since it is difficult to find analytic solution to the above optimal problem, we solve it numerically by the ‘interior point’ algorithm in MATLAB. This strategy will be called ‘dynamic’.

4.2 Performance Analysis

Performance and risk measures of the dynamic strategy are shown in Table 10&11, Figure 8. With constantly adjusting our position, dynamic strategy notably outperforms others.

Table 10 Comparison of Dynamic Strategy with Others

	Mean Return	Standard Deviation	Cumulative Return	Sharpe Ratio
Index	0.1081	0.2438	1.7963	0.4433
Strat 1	0.0786	0.2438	1.4606	0.3225
Strat 2	0.1098	0.2890	1.6636	0.3762
Strat 3	0.0458	0.1518	1.3197	0.3018
Strat 4	0.1060	0.1386	2.0423	0.7650
Dynamic	0.1222	0.0910	2.3763	1.3425

Table 11 Comparison of Dynamic Strategy with Others

	VaR	CVaR	Max DD
Index	-7.0182	-11.5461	0.5483
Strat 1	-6.7935	-11.1503	0.3510
Strat 2	-6.8205	-12.6678	0.4253
Strat 3	-3.1797	-4.9735	0.2623
Strat 4	-2.3453	-3.7050	0.2249
Dynamic	-1.0549	-2.4075	0.0758

Figure 8 Cumulative Return of Dynamic and Other Portfolios

4.3 Scenario Analysis

In order to understand what the dynamic strategy does and why it makes sense, we make further analysis on positions (weight of the four portfolios) of dynamic portfolio over time. Before that, we need to have a deeper understanding on our dynamic strategy’s motivations: performance of our four portfolios under different scenarios.

From Table 12&13 and Figure 9&10, we can find that after properly scaling, Strategy 1 and Strategy 2 are similar while Strategy 3 and Strategy 4 are similar. Theoretically, the value of at-the-money call or put is linear to volatility. During bear market like scenario 1 (see Figure 9), realized volatility is higher than implied one, so option strategies will yield higher return than normal time. Inversely, during bull market like scenario 2 (see Figure 10), realized volatility is lower than implied volatility, then index momentum strategy will outperform option momentum strategies, where straddle strategies might even produce a negative sharp ratio.

However, these patterns cannot be employed directly into our mean-variance frame work (dynamic strategy) due to concerns about overfitting. Also, there are some situations where stock market is oscillating, making it hard to correctly define it as bull or bear market. The differences of sharp ratio between Strategy 1 vs 2, Strategy 3 vs 4, may offer an intuitive motivation of dynamic strategy. During bear market, Strategy 2 dominates Strategy 1 and Strategy 4 dominates Strategy 3, vice versa during bull market.

Table 12 Bear Market Comparison (01/03/2008 - 03/06/2009)

	Sharpe Ratio	Wealth	Max DD
Strat 1	0.1912	1.0562	0.3510
Strat 2	0.6786	1.1566	0.4253
Strat 3	1.1701	1.1471	0.2623
Strat 4	1.5972	1.1884	0.2249

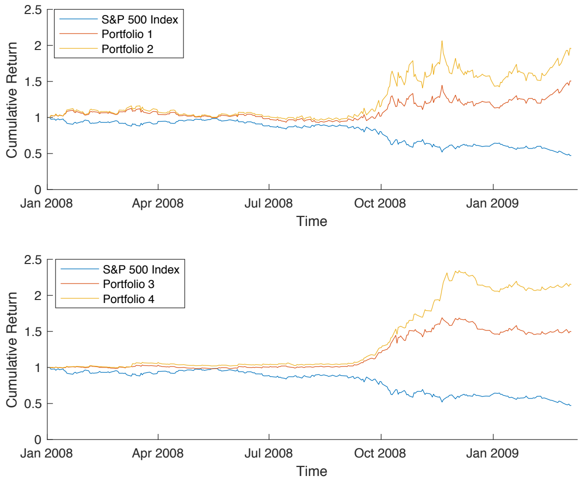

Figure 9 Bear Market Comparison (01/03/2008 - 03/06/2009)

Table 13 Bull Market Comparison (12/31/2012 - 10/14/2014)

	Sharpe Ratio	Wealth	Max DD
Strat 1	0.9515	1.1806	0.3510
Strat 2	0.7761	1.1140	0.4253
Strat 3	-0.8438	0.8979	0.2623
Strat 4	-1.3144	0.8801	0.2249

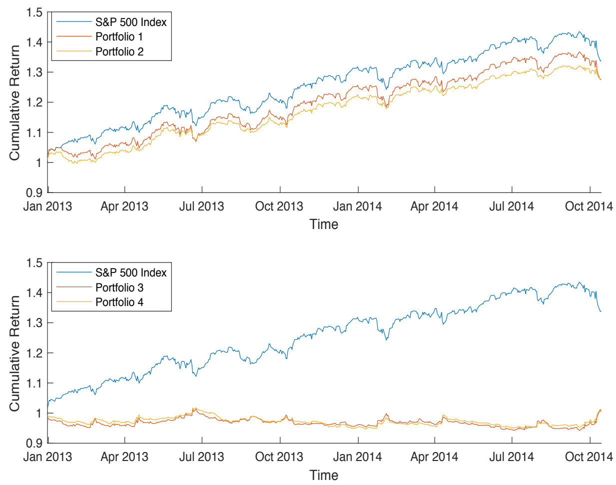

Figure 10 Bull Market Comparison (12/31/2012 - 10/14/2014)

4.4 Weight Analysis

Figure 11 shows the rounded position of our dynamic portfolio, 1 means long and -1 mean short. (however, actual weights are decided by our model.)

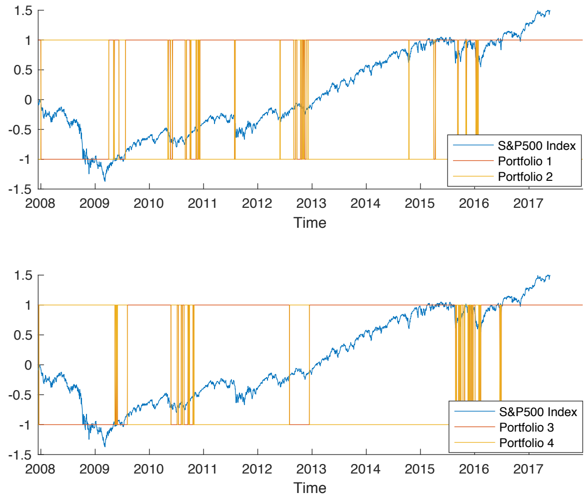

Figure 11 Positions in Strategy 1&2 and 3&4

Reasonably, dynamic portfolio always holds opposite position for Strategy 1 & Strategy 2 and for Strategy 3 & Strategy 4. The intrinsic motivation for the first pair of opposite position is to create a delta-hedge portfolio, trying to maximize the profit from gamma and avoid loses from other Greeks.

The intrinsic motivation for the second opposite position is to enter a calendar spread. During a 90-day period, we use majority of days to train our model, and enter calendar spread in a proper way at the end of this period. It makes sense, since we know that Strategy 4 has similar performance with Strategy 3 for 60 days, and difference between them varies from 60 to 90 days. (see Figure 12) As a result, the dynamic strategy reduces the risk by risk diversification and correct prediction of positions of a calendar spread. Therefore, dynamic portfolio produces a very good sharp ratio, wealth with an extremely low maximum drawdown.

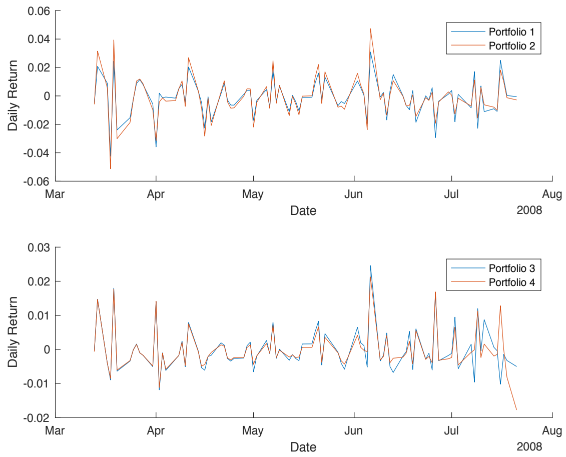

Figure 12 Daily Return of Strategy 1&2 and 3&4

5. Conclusion

This paper sets out to investigate the effectiveness of capturing momentum effect with options while minimizing market risk. By introducing adjustable scaling factors, the underleveraged option portfolios can utilize momentum effect and simultaneously to maintain a lower risk than directly investing in index. In this process, additional information from the option Greeks is exploited to balance the return and risk.

Upon observing many large drawdowns in our result, we fine-tune our strategy by continuously monitoring maximum drawdowns and quit the market when appropriate. From our back-testing, all four portfolios show a greater profitability under such modification.

In addition, the negative correlations between our strategies and the market index indicate a hedging opportunity. By buying index alongside our strategies, a better tradeoff between profit and risk is realized.

As an extension, we use MA (moving average) crossover to determine whether we should long/short straddle to long/short volatility. Having a lower drawdown, the profit of such a strategy is notably high.

The fact that the dominant portfolio switches in different market scenario implies that mean-variance framework could be applied to construct a 'dynamic' portfolio comprised of the four original portfolios. After analyzing the optimized weight given by the strategy, we realize that it automatically extracts a low risk profit by pairing Strategy 1&2 and Strategy 3&4. As a result, the dynamic portfolio achieves a remarkably stable return curve.

Based on our work, more researches could be done to explore the possibility of using other derivatives to profit from momentum effects.

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