CDS Risk Modeling and Hedging Strategy: A Statistical Approach

Chapter 1

Abstract

In this paper, we argue that a key component of mitigating market risk, counterparty risk, and moral hazard would be determining a correct level of capital reserves. To support this, we first implement a hazard model, where the analysis on the ABX AAA 2006-2 index shows that the market actually diverges significantly from fundamentals, resulting in significant mark-to-market risk. We then further propose a factor-based model which describes its deviation from the fundamental price. Next, through the Kupiec test, we show that the VaR distribution likely follows a Student-t Distribution. We further utilize a hidden Markov regime switching approach to determine an appropriate stressed period. In the last chapter, we present a regression-based hedging strategy that effectively mitigates the risk of holding a CDS contract.

Chapter 2

Introduction

On September 16th, 2008, after experiencing substantial losses on a large portfolio of subprime CDS, AIG received access to an $85 billion revolving credit facility authorized by the Federal Reserve Board. The rescue of AIG and its counterparties, which when combined with other eventual government facilities would total over $120 billion [1], sent a message to financial markets that large institutions with significant derivative exposures would not be allowed to fail. This action has contributed to a sense among practitioners that when financial conditions deteriorate into a crisis, the government will ensure that your derivative exposures to large institutions be made whole.

By rescuing the counterparties of AIG in 2008, the Federal Reserve has, to some extent, removed or reduced the financial incentive a practitioner has to fully evaluate the credit quality of a potential counterparty. If a market participant believes in a continuation of this crisis policy, he or she might be more likely to enter into a contract with a large, rescue-worthy financial institution as opposed to a smaller institution whose failure would be inconsequential to the health of financial markets as a whole. In addition, this removes or reduces incentives that large financial institutions had to minimize risk in order to attract additional or continuing derivatives business. In order to reintroduce the necessary financial incentives behind properly assessing counterparty risk, the government needs to develop a credible strategy to handle institutions whose failures it believes would have severe market implications.

In this paper, we argue that a correct level of capital reserve would be the key solution to all these problems: Market risk, Counterparty risk, and moral hazard. With a sufficient reserve, AIG would be able to avoid a severe capital requirement as a result of being downgraded, and its counterparty would able to cover the loss with a much smaller chance of causing a systematic crisis. Moreover, accurate capital requirements could allow the poorly managed banks to fail without dragging the whole system down. As such, it helps to avoid the infamous moral hazard problem known as “Too Big to Fail”.

To simulate the reserve, we propose both a hazard rate model and a factor based VaR model. Due to the mark-to-market accounting requirement, we focus on the analysis of the ABX.HE AAA index, which is the most widely used index for mark-to-market CDO and CDS products. In the mid 2000’s, Markit introduced a vehicles through which one could monitor the overall performance of the subprime MBS market, known as the ABX indices [2]. By combining specific tranches of 20 large subprime ABS deals completed within a certain time frame, each index was designed to track the performance of these tranches over time. This ability to write derivative contracts based on the ABX allows for both hedging and speculating in a market for which otherwise would be difficult to do on an instrument-by-instrument basis.

In the following chapters, we will demonstrate how an investor can calculate their appropriate reserve based on this model. Also, the investor can easily extend the model to the calculation of the reserve for counterparty risk. In the end, we will also show a hedging mechanism, which could help investors efficiently reduce their market risk.

Chapter 3

Pricing the Bond Value and Reserve

3.1 ABX Modeling Analysis

Modeling ABS can be difficult since its price is driven by several interacting factors: the interest rate process, prepayment behavior, and default behavior. We, therefore, adopt a Monte Carlo simulation to price the ABX.HE, such that we can model the following processes separately:

Term structure dynamics: The interest rate process is assumed to follow the Hull-White model, where the short rate follows an Ornstein-Uhlenbeck process under risk-neutral measure with a time-varying mean:

$$dr_t = [\theta(t) - \kappa r_t]dt + \sigma W_t$$

We first fit the term structure using the pure discount bond prices, and then calibrate the parameters κ and σ using caplet volatilities. Once κ and σ are calibrated, $\theta(t)$ can be implied according to Veronesi (2010)[3].

$$\theta(t) = \frac{\partial f(0,t)}{\partial t} + \kappa f(0,t) + \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa t})$$

$$f(0,t) = -\frac{\partial logZ(t)}{\partial t}$$

Hazard estimates of prepayment and default: We adopt a reduced-form model to depict prepayment and default behaviors, assuming that the two behaviors are driven by the following exogenous factors:

- Seasonality (i.e. summer month indicator)
- Coupon gap, which is difference between the tranche coupon and 3-month lagged 10-year LIBOR.
- Loan-to-value (LTV), which is the ratio between the remaining balance of the bond and the house price. The house price dynamic is assumed to be a Geometric Brownian Motion with an independent stochastic driver.

$$\frac{dH_t}{H_t} = [r_t - q_H]dt + \Psi_H dW_{H.t}$$

The hazard rates are estimated under Schwartz and Torous (1989) [4] log-logistic proportional hazard

$$\lambda(t) = \lambda_0(t)e^{\beta_1 v_1 + \beta_2 v_2 + \beta_3 v_3 + \cdots} dt + \Psi_H dW_{H.t}$$

$$\lambda_0(t) = \frac{\gamma p(\gamma t)^{p-1}}{1 + (\gamma p)^p}$$

Here we adopt the coefficients (γ, p, β_1, β_2, and β_3) from the Stanton and Wallace (2011) paper.[5] Having obtained the hazard estimates and interest rate dynamics, we can obtain the monthly default and prepayment behavior, then allocate the cash according to the seniority and the balance of each tranche. Details of the waterfall rule and the monthly balance of each tranche can be found on www.markit.com and on Bloomberg.

3.2 ABX Simulation Result

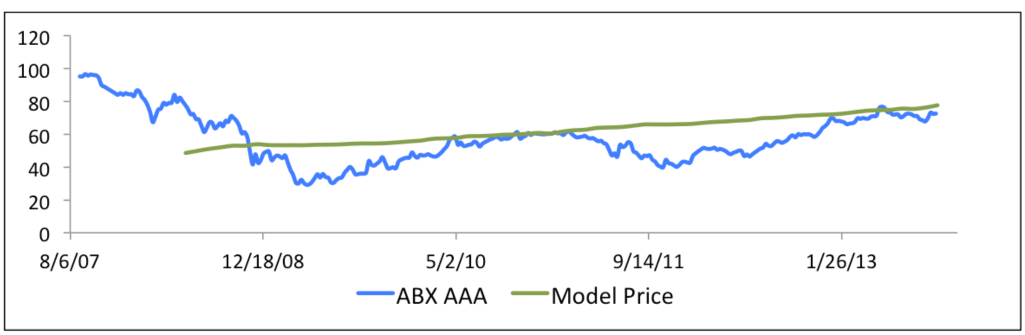

Figure 3-1. ABX.HE 2006-2 AAA Market Price vs. Model Price

In this sector, we examine the pricing model on ABX.HE 2006-2. The monthly balance of each tranche and weekly index price are obtained from Bloomberg. The result is shown in Figure 3-1. In performing the valuation from 2008/7-2013/10, we can see that in the long term, the market price tends to converge to its fundamental price. The ABX.HE index CDS seems to be systematically mispriced. The large up-front payments for the CDS are not consistent with the hazard rate model and are likely explained by the other risk factors.

This finding contributes to a growing concern linked to the recent passage of the Volcker rule. Under this new rule, some CDOs are required to be reclassified as "Available for Sale," and therefore banks must mark-to-market these portfolios. According to Stanton and Wallace's (2011) [5] calculation, if we mark-to-market all bonds rated AAA on Jun 30, 2009, the loss would be $90.8 billion. As a result, investors must be aware of their mark-to-market risk, as it may affect their regulatory capital requirements and the timing of liquidations. In the coming section, we will build up a factor model, to explain the excessive fluctuation of the bond beyond its fundamental value.

3.3 Reserve and VaR Model

To determine a sufficient capital reserve, we start by following Basel III, which strictly defines a standard requirement of market risk capital. Here, banks are asked to calculate 10 day VaR at a 99% confidence level on a daily basis. In addition, they must also calculate a "Stressed Value at Risk measure" with model input based on a 250-day period of Stressed Regime.

$$C = max\{VaR_{t-1}, 3 * VaR_{avg}\} + max\{sVaR_{t-1}, 3 * sVaR_{avg}\}$$

In this paper, we adopt a factor-based approach to calculate VaR, one consistent with standard practices in the financial sector. The idea is to map each asset of the portfolio to common risk factors and base the calculation of VaR on these risk factors. Mathematically, we model the daily return of an asset as a linear function of factor returns and residuals as follows.

$$r = \sum_i \beta_i f_i + \epsilon$$

The portfolio variance can be calculated as:

$$\sigma^2 = \beta^T F \beta + \sigma_\epsilon$$

Here, r is the asset return, f_i is the factor return, ϵ is the idiosyncratic risk, and F is the variance-covariance matrix of the factor returns. We include six risk factors for calculating VaR. First, we use the S&P 500 Index, which captures the systematic risk of the financial market. Second, we use the LIBOR-OIS, a "liquidity spread" defined as the difference between the three-month Libor and the three-month overnight index swap, measuring short term risk. Third to fifth, we use the 5Y and 10Y Swap Rate, which indicates the change of yield curve as well as the 30Y Mortgage Rate. Lastly, we use the CDX IG spread and the implied volatility of 3M10Y swaption.

Basel III also requires that each VaR model captures the nonlinearities beyond those inherent in options and other relevant products. Therefore, we also include second-order terms of risk factors.

3.4 VaR Simulation Results

Accordingly with Basel III, we perform Monte Carlo simulations as a VaR assessment tool. To do this, we first calculate the covariance across the market risk factors. We then generate random routes by assuming the factor movements follow a normal distribution, a Student-t distribution, and a GARCH(1,1) model. Next, we calculate the asset price accordingly. After a repeated series of simulations, we realize a simulated distribution of our portfolio and obtain our VaR.

To assess the performance of our one-day horizon VaR forecasts using the rolling window procedure, we start with a 100-day window. With this data, we can calculate a one-day forward VaR for each of these 3 methods. By using this rolling window, we can assess the historical probability that the realized return is less than our 99% VaR.

Table 3-1. ABX Hot-Run AAA Out of Sample Back-testing

	Historical Prob.	Kupiec Test	Reject
Linear Model (Normal)	2.16%	10.29	Yes
Linear Model (Student T)	1.44%	1.45	No
Quadratic Model (Normal)	2.16%	10.29	Yes
Quadratic Model (Student T)	1.44%	1.45	No
GARCH(1,1)	1.98%	7.51	Yes

Table 3-2. ABX Hot-Run AA Out of Sample Back-testing

	Historical Prob.	Kupiec Test	Reject
Linear Model (Normal)	2.52%	16.82	Yes
Linear Model (Student T)	0.90%	-0.06	No
Quadratic Model (Normal)	2.52%	16.82	Yes
Quadratic Model (Student T)	0.90%	-0.06	No
GARCH(1,1)	2.52%	16.82	Yes

Next, we use a Kupiec test [6] to test the null hypothesis that our VaR historically indicates the 99% level. The test statistics are shown as follows: LR is the log likelihood ratio, T is the number of observation points, N is the number of return below VaR, and p is the probability for a corresponding confidence level. In our case, p is equal to 1%. Since this test statistic is asymptotically χ^2 distributed with one degree of freedom, the confidence level is 3.84 at $\alpha = 5\%$.

$$LR = -2ln\{(1-p)^{T-N}p^N\} + 2ln\left\{\left(1-\left(\frac{N}{T}\right)\right)^T \left(\frac{N}{T}\right)^N\right\}$$

We found that the factor VaR model with Student-t distribution does a satisfactory job of describing the VaR distribution. In this model, we fail to reject the null hypothesis that p=1% is the true possibility, showing that the model is statistically significant. On the other hand, adding quadratic term and using a GARCH(1,1) model does not statistically significantly improve the VaR prediction out of the sample.

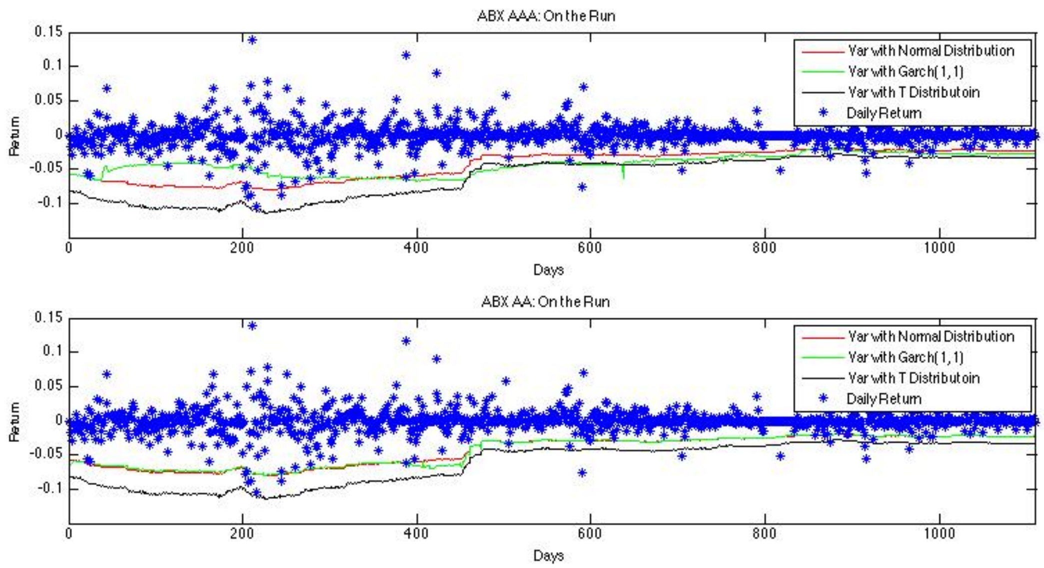

Figure 3-2. Out of the Sample VaR Back-Testing

3.5 Identifying Stressed Periods: Hidden Markov Regime Switching Analysis

The last step of accessing stressed Value at Risk is to identify the stressed periods. Several ideas and methodologies have been proposed to distinguish among different regimes. For example, distinguished regimes have been based on the volatility level of the overall market measured through the VIX, Principal Components Analysis, and other methods. The majority of those methodologies use an arbitrary threshold to distinguish among two regimes. A naïve example would be to define a critical level for the VIX (30%) and implement a strategy following that specific rule. Those methodologies are not objective, and even worse, they are extremely prone to data-mining and overconfidence from in-sample results.

Instead of choosing this threshold artificially, we introduce a hidden Markov regime switching approach [7] to determine the appropriate stressed period. The basic idea is that regimes exist both where assets perform very well and where they perform very poorly. Therefore, instead of using arbitrarily defined thresholds to define the regimes, we use maximum likelihood estimation to define the parameters in different regimes, as well as a Markovian transition matrix which describes the evolution of the system. A clear description of the methodology and an application has been given by Kritzman et al (2012). [8]

Correlation is a key input when calculating all of the risk measures, and therefore assessing correlation accurately is extremely important for a financial institution. In this paper, the Hidden Markov Regime Switching technique is used to find different covariance matrices in different regimes instead of the classical application of finding expected returns in different regimes. We use Libor-OIS as the benchmark, and perform this method to find the 250-day period which has the highest likelihood of being in the high correlated regime. We denote the coveariance matrices in two regions by σ_1 and σ_2, the Markovian transition matrix as M, and initial probality of being in the first regime as p_1.

To find the Maximum Likelihood parameters, we used the Baum and Welch algorithm. The result is demonstrated in the following chart. This method gave us a well-defined stressed region from 2008/9/3~2009/9/1. Table 3-3 is an example of the capital reserve as of 2012/5/1. Again, we can see that using the Student-t distribution, the tail risks are better captured.

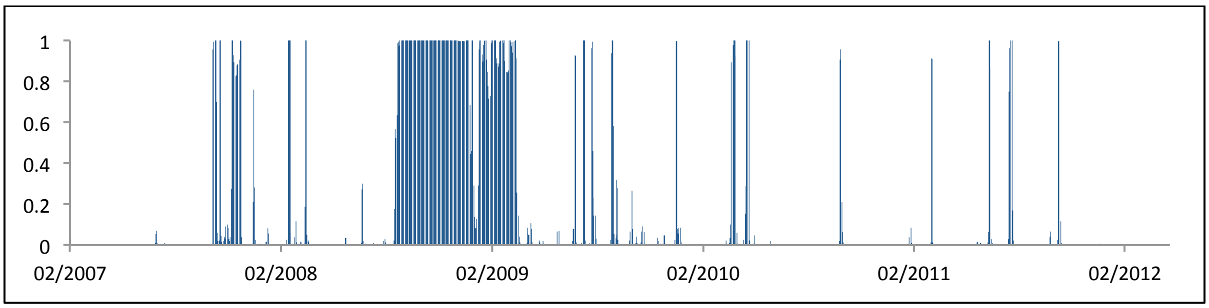

Figure 3-3. Likelihood of being in the High Correlation Region

Table 3-3. Calculated ABX AAA Capital Reserved as of 2012/5/1

	sVaR(avg)	VaR(t-1)	VaR(avg)	Reserve*
Linear Model (Normal)	3.60%	2.30%	2.20%	18.34%
Linear Model (Student T)	5.11%	3.22%	3.18%	26.22%

* Reserve = 3* 10days-sVaR(avg) + max{10days-VaR(t-1), 10days-VaR(avg)}
* 10 days-VaR= 1day-VaR*sqrt(10)

3.6 Capital Reserves for Counterparty Risk

Combining the hazard rate model and the factor-based VaR model, we should be able to come up with an appropriate level of capital reserve for counterparty risk. According to Basel III, Bank can model CVA VaR by the following Equation:

$$\mathrm{CVA} = \mathrm{LGD_{mkt}} \sum_{i=1}^{T} max\left(0, exp\left(-\frac{s_{i-1}t_{i-1}}{LGD_{mkt}}\right) - exp\left(-\frac{s_i t_i}{LGD_{mkt}}\right)\right)\left(\frac{EE_{i-1}B_{i-1} + EE_i B_i}{2}\right)$$

Here EE_i is expected exposure at time i, which can be calculated using the hazard rate model. Accordingly, s_i is the credit spread of the counterparty, whose VaR can be obtained by using factor-based VaR model for the bond or CDS. Similar to the capital charge for market risks, banks are also required to calculate both a normal and stressed CVA-VaR, then multiply this result by a factor of 3 in order to get the counter party risk charge. The calculation follows exactly the same as we have done in the above section.

In this chapter, we have introduced all the necessary tools to determine to quantify both the mortgage CDS price and the VaR. We also showed that these models provide significant VaR predictive performance. Using these tools, investors can determine the appropriate reserves for both writers of the CDS and for buyers of the CDS such that they can manage their risk more easily. In the next chapter, we would like to analyze a method to hedge CDS instruments.

Chapter 4

Hedging the CDS Instruments

In our previous sections, we attributed changes in the ABX.HE price to a factor based approach. Here to hedge, we denote a market risk factor (m), an industry risk factor (I), and a residual risk factor (r). To hedge, we seek a portfolio of proxy assets, emulating the same movements according to similar risk factors. To hedge the market risk, we assume we have access to an S&P tracking portfolio, which we refer to by the ticker SPX. Next, in an attempt to proxy some of the industry risk, we include the FTSE NAREIT Mortgage REIT total return index as well as the BBREMTG Mortgage REITs index.

To determine the quantities of each proxy we need in the portfolio, we assume that changes in the CDS prices are related to the price of the CDS, the prices of the proxies, and the changes in the prices of the proxies. Mathematically speaking:

$$\Delta \mathrm{CDS_t} = f(\mathrm{CDS_t}, \mathrm{Proxy_{1,t}}, \ldots, \mathrm{Proxy_{n,t}}, \Delta\mathrm{Proxy_{1,t}}, \ldots, \Delta\mathrm{Proxy_{n,t}})$$

We assume the following functional form such that we can estimate relative hedging quantities (β), through a linear regression framework:

$$\Delta \mathrm{CDS_t} = \mathrm{CDS_t} \sum_{i=1}^{n} \beta_i \frac{\Delta \mathrm{Proxy_{i,t}}}{\mathrm{Proxy_{i,t}}} + \varepsilon_t$$

In the linear regression, we effectively regress CDS returns given proxy returns to estimate the relative hedging quantities (β):

$$\frac{\Delta \mathrm{CDS_t}}{\mathrm{CDS_t}} = \sum_{i=1}^{n} \beta_i \frac{\Delta \mathrm{Proxy_{i,t}}}{\mathrm{Proxy_{i,t}}} + \varepsilon'_t$$

We then rearrange to find the actual hedging quantities (h) as:

$$h_i = \beta_i \frac{\mathrm{CDS_t}}{\mathrm{Proxy_{i,t}}}$$

To evaluate our hedging performance, we simulate the cost of building a replicating portfolio, rebalanced weekly. After determining which quantities of the proxy assets need to be purchased, we fund the difference required to match the CDS price. We introduce a leakage term, which approximates the cost of the hedge. This is calculated as the cumulative funding corrections plus the funding required to purchase the proxies, accrued each day by the interest rate. We additionally add transaction costs which are calculated as 0.1% of the notional traded through funding bonds or asset purchases.

Unfortunately, this initial approach was not successful. We attribute this to three likely reasons. First, the CDS is likely non-linearly related to our hedging proxies. For example, the expected relative changes in CDS price may change according to its price. Second, our hedging estimation requires variance in both the CDS and our proxies. During the time prior to 2008, the CDS shows very little variance, and thus makes it difficult to get an accurate hedging quantity estimate. Third, we are finding quantity estimates cross-sectionally, while it may be the case that either the CDS or the proxies have different response times to shocks in the risk factors.

To alleviate the first problem of non-linearities, we note that locally in time, the CDS sensitivity to our proxies should remain effectively stable. Therefore, our hedging parameters estimated within this local window should be more reliable at the current CDS price, given enough data. To exploit this, we additionally estimate hedging quantities with an exponentially weighted regression in time. We choose a half-life of 20 days, such that squared residuals 20 days ago penalize the fitted parameters half as much as today.

We find that this provides a noticeable improvement to our hedging performance, however, still leaves plenty of room for further improvement. Mainly, the hedge still does a poor job during late 2007, when losses in the CDS market occur weeks before big drops in the FTSE NAREIT Mortgage REIT Index and the BBREMTG Index. Our hedging strategy will not take full advantage of this lagged reaction, as we are estimating the hedging ratios cross-sectionally, and perhaps not modeling the full co-dependencies that may exist.

We additionally attempt a hedge under the assumption that the within a reasonably window of time, the sensitivities of each asset to the risk factors is approximately constant or in other words, the CDS and proxies are effectively locally co-integrated. Then, through a Taylor Series approximation:

$$\mathrm{CDS_t} = \mathrm{CDS_0} + \sum_{j=0}^{j=t} \Delta\mathrm{CDS_j} \approx \mathrm{CDS_0} + \sum_{j=0}^{j=t}\left(\frac{\partial \mathrm{CDS_j}}{\partial \mathrm{m}}\Delta\mathrm{m} + \frac{\partial \mathrm{CDS_j}}{\partial \mathrm{I}}\Delta\mathrm{I} + \frac{\partial \mathrm{CDS_j}}{\partial \mathrm{r}}\Delta\mathrm{r}\right)$$

$$\mathrm{Proxy_t} = \approx \mathrm{Proxy_{i,0}} + \sum_{j=0}^{j=t}\left(\frac{\partial \mathrm{Proxy_j}}{\partial \mathrm{m}}\Delta\mathrm{m} + \frac{\partial \mathrm{Proxy_j}}{\partial \mathrm{I}}\Delta\mathrm{I} + \frac{\partial \mathrm{Proxy_j}}{\partial \mathrm{r}}\Delta\mathrm{r}\right)$$

Under this assumption, we estimate the hedging quantities using our exponentially weighted regression using a 20 day half-life:

$$\mathrm{CDS_t} = \beta_0 + \beta_1 \mathrm{FNMRTR_t} + \beta_2 \mathrm{BBREMTG_t} + \beta_3 \mathrm{SPX_t} + \varepsilon_t \ (2)$$

We find this method provides superior hedging performance, although it dips very early in late 2007, once the proxies similarly feel a shock a few weeks later, the hedge seems to do well in mitigating much of the loss over time.

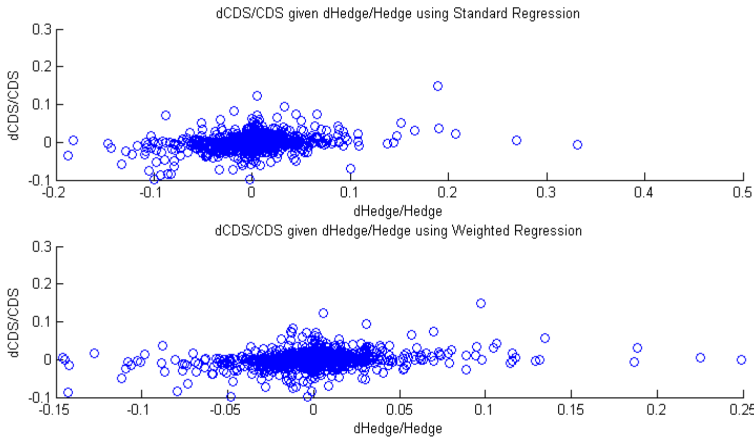

Figure 4-1. Hedging Residuals using equation (2) with both standard regression and weighted regression.

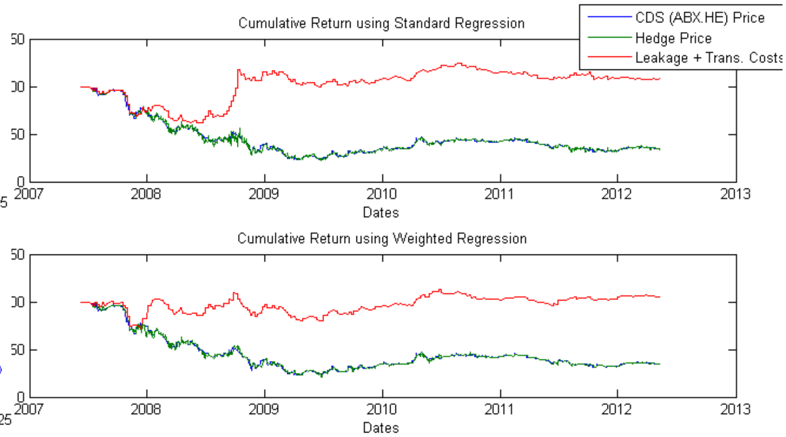

Figure 4-2. Cumulative hedging leakage using equation (2 with both the CDS and Hedge Portfolio returns, re-balanced weekly.

Chapter 5

Summary

In this paper, we first proposed a hazard rate model to simulate the fundamental CDS price. This analysis on the ABX AAA 2006-2 index shows that the market actually diverges significantly from fundamentals. This echoes the concern that mark-to-market accounting may exacerbate the severity of a financial crisis. As a result, investors must be aware of their mark-to-market risk, as it may affect their regulatory capital requirements and the timing of liquidations.

To estimate the correct level of capital reserves, we built up a factor model which may explain the excessive fluctuation of the bond beyond its fundamental value. We performed a Kupiec test to access the model's predictive performance out of the sample. Our results show that if we assume a normal distribution of the factor returns, the model systematically underestimates the level of VaR. However, we found that the factor VaR model with Student-t distribution does a satisfactory job of describing the VaR distribution. Surprisingly, while adding a quadratic term and using a GARCH(1,1) model better explains the market movement in-sample, it did not improve the VaR prediction out-of-sample in a way that is statistically significant.

In order to define the stressed period unambiguously, we introduce a hidden Markov regime switching approach. The model picked 2008/09/03~2009/09/01 as the most stressed 250-day periods without any arbitrary threshold to distinguish among different regimes. With these tools, investors can easily calculate the appropriate level of reserve. At last, we proposed a regression-based hedging model, which proved be effective in hedging CDS instruments.

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